ASSESSMENT OF THE EXCHANGE RATE RISK EXPOSURE IN TUNISIA'S EXTERNAL PUBLIC DEBT PORTFOLIO: A DELTA-NORMAL VAR APPROACH IN THE CONTEXT OF SUSTAINABLE FINANCE DEVELOPMENT

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Abstract

This paper assesses the exchange rate risk exposure of Tunisia's external public debt portfolio using the delta-normal Value at Risk (VaR) approach. Based on daily data from 2004 to 2019, focusing on the main borrowing currencies (the euro, US dollar, and Japanese yen), the study identifies the riskiest currencies and offers policy recommendations. The findings highlight significant exposure to the Japanese yen, while the US dollar appears to act as a hedge against currency volatility. The research underscores the importance of adjusting the portfolio structure based on currency risk profiles and Tunisia's trade dynamics. This analysis contributes to the broader objective of sustainable public finance development by promoting more resilient and responsible debt management practices.

Keywords: debt management, currency risk, financial stability, portfolio optimisation

JEL Classification: F31, F34, C53

1. Introduction

In an increasingly environmentally and sustainability-conscious world, our research fits within the context of sustainable finance. While our study does not exclusively focus on ecological aspects, it positions itself within a broader consideration of financial sustainability. Sustainable finance aims to integrate responsible, social, and environmental financial practices into portfolio and risk management. In this regard, the analysis of exchange rate risk in Tunisia's external public debt is relevant within the perspective of economic stability and

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prudent public finance management, aligning with the concerns of sustainable finance. Proper management of external public debt plays a crucial role in advancing sustainable finance. Indeed, effective management of external debt has the potential to enhance a country's financial stability, creating a conducive environment for investment and economic growth, both fundamental aspects of sustainable finance. Additionally, proficient debt management grants countries improved sovereign ratings, thus facilitating better access to sustainable capital markets. This, in turn, allows them to issue green or social bonds to finance environmentally friendly and socially responsible projects.

It is worth noting that sustainable development fundamentally hinges on poverty reduction while preserving scarce resources. Prudent debt management offers the opportunity to seek external funding to support poverty alleviation programs, education, healthcare, clean water supply, and the deployment of renewable energy, all of which are major objectives of the Sustainable Development Goals (SDGs). Appropriate management of external public debt can play a vital role in preventing the accumulation of unsustainable debt, which could hinder the capacity to finance long-term sustainable initiatives.

The management of external public debt and the development of sustainable finance are closely intertwined. Through careful and responsible debt management, it can serve as a catalyst for environmentally and socially responsible investments, thereby contributing to progress towards a more sustainable future.

The evaluation of the exposure to exchange rate risk associated with Tunisia's external public debt portfolio is driven by several key factors. Firstly, a deep understanding of this exposure is fundamental for responsible financial management of public debt, thereby preventing potential financial difficulties arising from unfavourable exchange rate fluctuations. Furthermore, assessing and managing exchange rate risk effectively allows for reducing the costs related to servicing public debt. This is of paramount importance for the prudent allocation of budgetary resources to other priority sectors. Indeed, cost reduction coupled with financial risk management forms the core of effective public debt management.

This evaluation will enable us to identify the currency with the least risk within the Tunisian external public debt portfolio. This identification will pave the way for recommendations to the relevant authorities regarding the optimal choice of currency for future borrowing.

In summary, the evaluation of exchange rate risk exposure is a crucial step to ensure responsible financial management, cost reduction, and mitigation of financial risks related to public debt, while contributing to informed decisions about currency borrowing in the future.

The uniqueness and the contribution of this study lie in its in-depth exploration of the underlying causes of the significant increase in Tunisian external public debt, in contrast to most previous works that have focused on the high rate of this debt and its negative implications for growth. One of the primary drivers of this increase is the depreciation of the Tunisian dinar against the major currencies comprising the external debt portfolio. Unlike previous studies, which often remained on the surface of this issue, our approach aims to delve deeper into comprehending the reasons behind this increased risk. This involves a thorough analysis of the factors contributing to the depreciation of the Tunisian dinar, as well as the impact of this trend on the evolution of public debt. By identifying these underlying causes, our work provides a more comprehensive and informed perspective on this complex matter.

This study stands out for its in-depth analysis of the reasons for the rise in Tunisian external public debt, with a particular focus on the depreciation of the Tunisian dinar against major currencies. It seeks to shed further light on the relationship between these factors, which is essential for formulating effective financial policies that can withstand exchange rate fluctuations. To achieve these objectives, the paper is structured as follows: after this introduction, Section 2 provides the literature review on both the exchange rate risk and the Value at Risk method. Section 3 describes the research methodology, and Section 4 presents and discusses the empirical results. Finally, Section 5 outlines policy recommendations and concludes the study.

2. Literature review

2.1. Exchange rate effect on external debt

Effective public debt management involves the art of mobilising low-cost borrowing resources while maintaining strict control over associated financial risks. Within this set of risks, we can identify credit risk, refinance vulnerability, liquidity exposure, interest rate uncertainty, and foreign exchange risk. For developing countries, the need to minimise risks can sometimes outweigh the simple reduction of debt-

related costs. In this regard, the World Bank, in its 2007 report on public debt management, highlights a crucial recommendation: it is imperative to clarify the paramount importance of risk reduction compared to cost savings when formulating a debt management strategy. This recommendation is based on observations showing that some sovereign defaults have been partially precipitated by governments focusing on short-term cost savings. An illustrative example is the issuance of substantial volumes of short-term debt denominated in foreign currencies, which has significantly exposed public finances to market conditions (World Bank, 2007).

The current work closely aligns with the guidelines set forth by the International Monetary Fund (IMF, 2001) regarding public debt management, whether for advanced, emerging, or developing nations. The chief aim of debt management is to diminish expenses and associated risks, directly benefiting the taxpayer. In the context of developing countries, one approach to achieving this involves the judicious selection of currencies for external borrowing.

Unlike developed economies, which benefit from vast and highly liquid financial markets and internationally convertible currencies, enabling them to issue foreign debt denominated in their domestic currency, developing countries face the constraint of borrowing primarily in foreign currencies. This situation largely arises from the frequently observed illiquid nature of their financial markets (Miller, 1997, as cited in Fisera et al, 2021, p.4).

In the literature, the Marshall-Lerner condition must be met for the depreciation of the exchange rate to benefit the country by stimulating export growth. However, this same exchange rate depreciation could have negative implications for the accumulation of the public debt stock, especially in the presence of a heavily indebted country. Thus, the repayment of loans in foreign currency becomes more costly in the presence of a depreciating currency. At that point, heavily indebted countries in foreign currencies must benefit from exchange rate appreciation episodes to repay the growing burden of their external debts.

In this context, a study conducted by Blessy (2019) sought to empirically examine the impact of exchange rate depreciation on external debt in a group of developing countries. Using quarterly data spanning from 2004 to 2017, the findings demonstrated that exchange rate depreciation tends to increase external indebtedness in the majority of the countries analysed. Additionally, Bouabidi (2023)

argues that the exchange rate emerges as one of the primary factors contributing to the phenomenal rise of external debt in Tunisia. More specifically, the author notes that 40% of the observed increase in Tunisia's external debt between 2009 and 2020 can be attributed to the exchange rate effect. In this perspective, the relationship between the exchange rate and external debt in emerging economies holds particular significance, given the adverse consequences that substantial exchange rate depreciation could entail. In this regard, Asonuma (2016) examined 18 instances of sovereign debt defaults and restructurings from 1998 to 2013. The author successfully established a positive link between real exchange rate depreciation and the emergence of debt-related crises. This relationship was explained by the fact that real depreciation increases the burden of external debt service, consequently raising the likelihood of a default situation.

2.2 Value at Risk

Several methods are available for studying the exchange rate risk faced by the external public debt portfolio. Among these approaches, the Value at Risk (VaR) method stands out. In reality, VaR assesses the most unfavourable anticipated loss for a portfolio over a given period, at a specified confidence level, and under market conditions considered to be normal (Jorion, 2006). In other words, VaR quantifies the maximum extent of losses to which a portfolio can be exposed during a specific period. Dowd (1998) defined VaR as the maximum amount one is at risk of losing on a given sum of money during a specific time frame, with a certain level of confidence. The duration of this holding period is typically one day, but can vary from a week to a month, a quarter, or even a year. However, the choice of this holding period significantly impacts the final VaR calculation: the longer the holding period, the more meaningful the VaR outcomes. Similarly, the choice of the confidence level, which acts as the threshold for defining the left tail of the portfolio's value distribution, directly influences the VaR outcome. As the confidence level increases, the value of VaR also increases. In this context, Dowd (1998) suggests that the choice of the confidence level depends on the purpose of calculating VaR. In reality, different confidence levels (such as 95%, 99%, or 99.5%) are used for various purposes. A lower confidence level is favoured for validation, while a higher confidence level is employed for risk management or determining capital requirements.

Therefore, there is no universally required holding period or confidence level when calculating VaR.

Blejer and Schumacher (1998) developed a comprehensive overview of VaR methodology for assessing the solvency of central banks and their exposure to risks. They highlighted that this approach, taking into account the balance sheet of monetary authorities, not only allows for the analysis of the origins of risks faced by central banks but can also help in anticipating financial crises. Nocetti (2006) adopted the same methodology as Blejer and Schumacher (1998) to examine early warning signals of the financial crisis that occurred in Argentina in 2001. Using VaR through Monte Carlo simulation, with a confidence level of 99.9% and 3 months, the author assessed that vulnerability indicators (calculated using VaR) provided a fairly accurate description of the crisis in Argentina.

Cakir and Raie (2007) applied VaR using both the delta-normal approach and Monte Carlo simulation to assess the impact of Sukuk, which are Islamic principles-based bonds, on the cost and risk structure of investment portfolios. Despite using a confidence level of 99% and a 5-business-day holding period, the results obtained from delta-normal VaR proved to be comparable to those derived from Monte Carlo simulation VaR. According to the analysis of these authors, the portfolio's VaR can be constructed by combining the risks associated with the underlying securities. In other words, it represents the envelope formed by the correlation and volatility between different risk variables over the study period.

In Tunisia, Ajili (2008) employed the VaR method using the delta-normal approach to assess the foreign exchange risk linked to the external public debt portfolio. She employed exchange rate data on a daily basis for the Tunisian Dinar against the three principal constituent currencies of the long-term public debt portfolio (Euro, US Dollar, and the Swiss Franc) for the period from January 1, 1999, to June 30, 2006. The results demonstrated that Tunisia preferred the Euro as the currency to manage exchange rate risk related to its external debt. However, breaking down the VaR analysis by component demonstrated that the Japanese Yen served as the primary origin of exchange rate risk for Tunisia.

Akbar and Chauveau (2009) confirm that the use of VaR and Cost at Risk (CaR) techniques in managing the public debt portfolios of developed countries is not a new practice. In fact, Ireland and Italy use VaR techniques to manage risks related to their debt portfolios,

while New Zealand employs both VaR and stop-loss limits approaches to manage exposure to exchange rate risks. These analyses are typically conducted with a confidence level of 95%.

The study conducted by Akbar and Chauveau (2009) aimed to evaluate and analyse the exchange rate risk associated with three currencies: the Euro, the US Dollar, and the Japanese Yen, on the portfolio of Pakistan's public debt using the VaR methodology over a daily holding period from 2001 to 2006. The conclusions of this study highlighted several significant findings:

- Annual exchange rate return series showed a better fit to a normal distribution compared to returns over the overall period (2001-2006);
- The results obtained from the Monte Carlo method and historical simulation were consistent with those from the Delta-Normal method;
- VaR calculated using the three methods showed a significant decrease in the potential maximum loss over the years, indicating improvements in exchange rate risk management;
- Beta analysis revealed that the US Dollar was the least risky currency as it had the only beta below one for all six years. In contrast, the Japanese Yen was the riskiest currency, with the highest beta throughout the period in the public debt portfolio;
- Despite being the individually least risky currency, the US Dollar contributed to presenting the highest risk as a component of VaR in some years, primarily due to its positive beta, which decreased significantly over the years, as well as its substantial weight in the portfolio.

In the same context of assessing exchange rate risk associated with the portfolio of public debt, Omrane (2012) also employed the VaR method to analyse the situation in Tunisia. Using daily exchange rate data for the Tunisian Dinar (TND) against the Euro (EUR), the US Dollar (USD), and the Japanese Yen (JPY), covering the period from 2 January 2004 to 31 December 2008, the results also confirmed that the Japanese Yen represents the riskiest currency, followed by the US Dollar. In contrast, the Euro appears to be the least risky currency compared to the other component currencies of the Tunisian external debt portfolio, corroborating the conclusions previously established by Ajili (2007) for the Tunisian context. Indeed, such results can be explained by the natural hedge provided by the Euro against exchange rate risk through export revenues denominated in this currency.

Virtually all of Tunisia's export revenues come from exports denominated in Euros.

In the continuation of our analysis, we will apply the VaR method to evaluate the associated risk with the exchange rates of the major foreign currencies that make up the portfolio of Tunisia's external public debt.

3. Research methodology

The purpose of this study is to evaluate the performance of managing Tunisia's external public debt in terms of exposure to exchange rate risk by identifying the risky currencies within the External Public Debt Portfolio in Tunisia (PDPET). To achieve this, the VaR approach is employed to assist the relevant authorities in pinpointing the currencies that pose the most risk compared to others in the portfolio (PDPET). Furthermore, this method will help determine the currencies that contribute the most to reducing the overall exposure to exchange rate risk in the PDPET.

The methodology adopted in this study is similar to that used by Ajili (2008) and Omrane (2012), but it extends over a longer period, from 02/01/2004 to 31/12/2019. This extension of the database highlights the significance of analysing fluctuations in exchange rate yield over the structure of Tunisia's external debt. It places particular emphasis on assessing the potential losses that this volatility could entail during the study period.

The first step in VaR calculation involves computing the returns for each exchange rate series for each year using geometric returns. We will then define the following three variables:

1/ $R_{EUR} = \ln \frac{\frac{EUR}{TND(t)}}{\frac{EUR}{TND(t-1)}} = \ln \frac{EUR}{TND_t} - \ln \frac{EUR}{TND_{t-1}}$

2/ $R_{USD} = \ln \frac{\frac{USD}{TND(t)}}{\frac{USD}{TND(t-1)}} = \ln \frac{USD}{TND_t} - \ln \frac{USD}{TND_{t-1}}$

3/ $R_{JPY} = \ln \frac{\frac{JPY}{TND(t)}}{\frac{JPY}{TND(t-1)}} = \ln \frac{JPY}{TND_t} - \ln \frac{JPY}{TND_{t-1}}$

Next, we proceed to calculate the portfolio's VaR of Tunisia's external public debt linked to exchange rate risk using the delta-normal version. We have chosen a confidence level of 95% and a one-day time horizon for this assessment. It is important to note that the application of this delta-normal approach, also referred to as the

variance-covariance method, is grounded on the assumption that the price fluctuations of the assets comprising the portfolio follow a normal distribution. In other words, the returns of the three exchange rates must follow a normal distribution for this method to be applicable.

The VaR of a portfolio with (n) assets can be computed using the VaR of each asset as follows:

$$VaR_n = -\beta\sigma_n Z = [VaR * M * VaR^T]^{1/2} \quad (1)$$

Where:

$\beta = 1.65$ quantile of the normal distribution at the 95% confidence level (1.65) (since the chosen confidence level in this study is 95%)

σ – the matrix of standard deviations (dimension: n x n)

Z – a vector (dimension: 1 x n)

M – the correlation matrix (dimension: n x n)

VaR^T – the transpose of the VaR vector

$VaR = [VaR_1, VaR_2, \dots, VaR_n]$

Thus, equation (1), which quantifies the maximum potential loss for a portfolio comprising (n) assets, is exposed, can be applied for our study (with a confidence level of 95%, the number of assets, n=3, and under the assumption of normality) as follows:

$$VaR_3 = 1.65Z_1\sigma_1\ 1.65Z_2\sigma_2\ 1.65Z_3\sigma_3 = [VaR * M * VaR^T]^{1/2}$$

$$VaR_3 = \left([1.65Z_1\sigma_1\ 1.65Z_2\sigma_2\ 1.65Z_3\sigma_3] \begin{bmatrix} 1 & M_{12} & M_{13} \\ M_{21} & 1 & M_{22} \\ M_{31} & M_{32} & 1 \end{bmatrix} \begin{bmatrix} 1.65 & z_1 & \sigma_1 \\ 1.65 & z_2 & \sigma_2 \\ 1{,}65 & z_3 & \sigma_3 \end{bmatrix} \right)^{1/2}$$

Where M_{ij} represents the correlation between the price movements of assets *i* and *j*.

In order to thoroughly study the VaR associated with the Tunisian external public debt portfolio, by applying the equation (1), we will begin by analysing the normality of the overall portfolio for the entire study period.

Here, the term “overall portfolio” refers to all the daily data concerning the three currencies (EUR, USD, JPY), specifically the exchange rate returns, covering the period from January 2, 2004, to December 31, 2019 (see Figure 1, in the Appendix).

According to the data presented in Table 1 (in the Appendix), which outlines the statistical properties of the returns of the three exchange rates, it is evident that these returns do not follow a normal distribution. The results of the Jarque-Bera normality tests show zero probabilities for all return series, confirming the rejection of the null hypothesis of normality (at a 5% significance level), implying that the return series do not follow a normal distribution.

Next, we calculated the correlation matrix (see Table 1a, in the Appendix) as well as the variance-covariance matrix (see Table 1b, in the Appendix) to analyse the correlation between the returns of the three exchange rates.

Faced with the issue of non-normality observed in the overall portfolio, we choose to address this situation by decomposing the initial portfolio into 14 distinct annual portfolios. Specifically, we opted for an annual decomposition of the overall portfolio to move closer to normality in the data (as demonstrated by the tests we applied to our data). The descriptive statistics affirm a more robust convergence of the annual portfolios towards a normal distribution as compared to the overall distribution.

Therefore, to proceed with the calculation of parametric VaR, it is necessary to verify the normality condition of the returns of the three exchange rates. Once this condition is met, the calculation of VaR involves several steps as follows:

1) Calculating the risk vector, the correlation matrix, and spotting the structure of the foreign public debt portfolio

a. The risk vector is calculated as follows:

$$V = \beta\ \sigma_i$$

$\beta = 1.65$ (for a 95% confidence level)

σ_i - the standard deviation of position *i*; σ - the vector of individual volatilities of the returns of the three exchange rates.

The risk vector can be rewritten:

$$V = \begin{pmatrix} 1.65\ \sigma_{EUR} \\ 1.65\ \sigma_{JPY} \\ 1.65\ \sigma_{USD} \end{pmatrix}$$

Analysing the different values (σ_i) associated with the annual returns of the trio of currency exchange rates throughout the observation duration may furnish valuable perspectives regarding the ranking of these exchange rates in terms of their volatility.

b. Calculating the correlation matrix

It is useful in determining the relationships between the three exchange rate returns of the three currencies. The correlation matrix (referred to below as M) can be rewritten as follows:

$$M = \begin{pmatrix} 1 & RJPY/EUR & RUSD/EUR \\ REUR/JPY & 1 & RUSD/EUR \\ REUR/USD & RJPY/USD & 1 \end{pmatrix}$$

c. Spotting the structure of the foreign public debt portfolio

The structure of the external public debt portfolio by currency under study can be represented by a vector A as follows:

$$A = \begin{pmatrix} A_i \\ A_j \\ A_z \end{pmatrix}$$

Where A_i , A_j and A_z - the flows of Tunisian external debt in the currencies *i*, *j*, *z*, respectively.

Thus:

$$A = \begin{pmatrix} A_{EUR} \\ A_{JPY} \\ A_{USD} \end{pmatrix}$$

According to the data published by the Central Bank of Tunisia[1], during the selected study period (from January 2004 to December 2019), the predominance of the Euro as the primary currency for Tunisia's external debt is significant. Indeed, the Euro represents an average of 58% of the total external public debt. The US Dollar follows with a share of 19%, and the Japanese Yen accounts for approximately 14%. In contrast, other currencies, such as the Kuwaiti Dinar, have a negligible share in the overall composition of Tunisia's external public debt. For example, the share of the Kuwaiti Dinar does not exceed an average of 4.6% during the analysis period. Therefore, the primary exchange rate risk associated with Tunisia's external public debt comes from the three major borrowing currencies: the Euro, the US Dollar, and the Japanese Yen. As a result, the exchange rate risk related to other currencies is considered insignificant.

We have opted to establish a portfolio consisting of external debt cash flows totalling 100 million TNDs. Consequently, from this 100 million TNDs of external debt, 91 million (58 million EUR, 19 million USD, and 14 million JPY) are subject to exchange rate risk, while the remaining 9 million do not carry any risk. Therefore, the vector A, which represents the flow of Tunisian external public debt, can be presented as follows:

$$A = \begin{pmatrix} 58 \\ 14 \\ 19 \end{pmatrix}$$

[1] *Data available at the website of the Central Bank of Tunisia, www.bct.gov.tn, accessed on 01/08/2023.*

The rationale behind selecting a constant vector of cash flows for the external public debt lies in the long-term stability of the structure of Tunisian public debt by borrowing currency.

2) Determining the risk matrix

This matrix reflects the level of risk associated with each borrowing currency and is formulated as follows:

$$(\beta\ V)'\ M\ (\beta\ V)$$

Where: M : the correlation matrix between the exchange rates;
V : the matrix of risks associated with each exchange rate;

Such that:

$$V = \begin{pmatrix} 1.65 \text{ б } REUR & 0 & 0 \\ 0 & 1.65 \text{ б } RJPY & 0 \\ 0 & 0 & 1.65 \text{ б } RUSD \end{pmatrix}$$

3) Multiplying the risk matrix by the vector A

Vector A signifies the average allocation of each currency within the external public debt portfolio, based on the following equation:

$$VaR = [A'\ (\beta V)'\ M\ (\beta V)A]^{1/2}$$

Where: A - the average allocation of each currency within the external public debt portfolio;
A' : transpose of A.
(βV)' : the transposed vector of volatilities multiplied by β.

For example, for the year 2004, the VaR associated with the annual portfolio is calculated as 0.2373293 million TNDs (see Table 3, in the Appendix). Interpreting this outcome, under a 95% confidence level, the Tunisian government's potential daily loss from a 100 million TND external public debt portfolio is limited to 0.2373293 million TND. This loss solely results from the exchange rate risk tied to the volatility of the three principal currencies included in Tunisia's external public debt portfolio.

4) Decomposition of VaR by currency

In order to mitigate the exchange rate risk linked to Tunisia's external public debt to the greatest extent possible, we conduct a currency-based decomposition of the VaR for the entire portfolio. The main objective of this decomposition is to determine the contribution of each asset to the total portfolio risk.

Therefore, the decomposed VaR can be calculated as follows:

$$VaR_i = \theta_i * A * VaR$$

Note that the assessment of the θi coefficient (as mentioned in Table 4 in the Appendix) reflects the impact of each risk factor relative to the total VaR of the portfolio.

5) Level (or degree) of VaR diversification

It entails comparing the total of individual VaR with the aggregate of decomposed VaR.

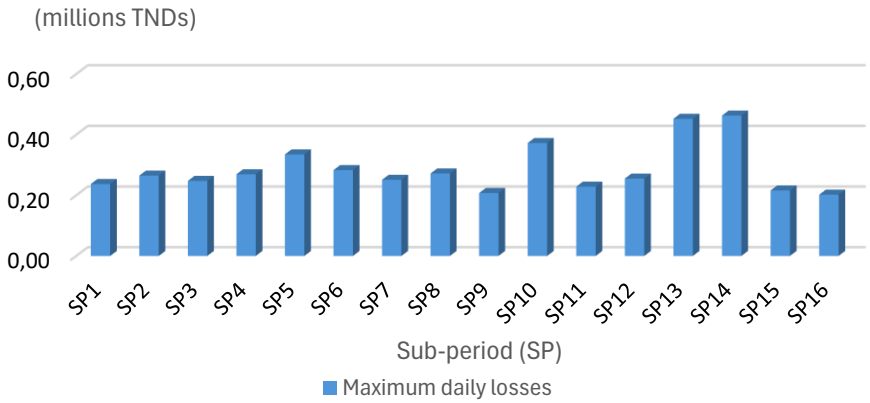
Finally, it should be noted that the five steps presented above will be repeated for each of the sixteen (16) annual portfolios.

4. Results and discussion

In our research, we chose to work with an annual length of the data series because we found that annual returns are closer to a normal distribution compared to the returns of the overall portfolio (see Table 2, in the Appendix). This implies that the parametric approach of VaR is more appropriate in this context, for a small developing country like Tunisia, given that it assumes that returns follow a normal distribution.

Graph 1

The calculation of VaR (in millions TNDs)



Source: Author's calculation based on matrix computation results.

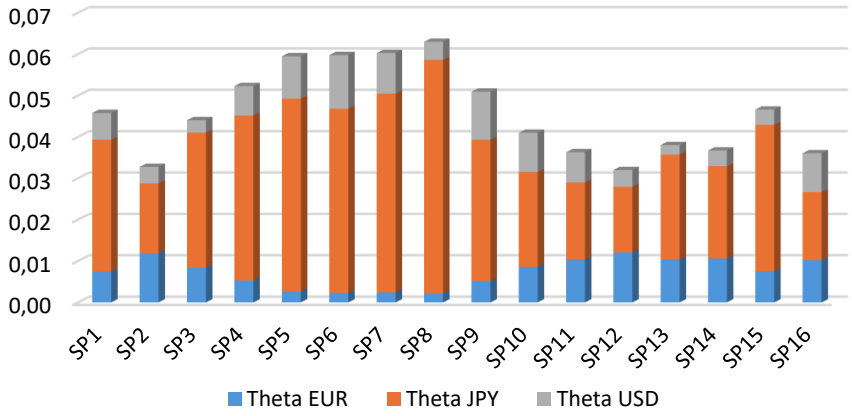
The analysis of the parametric VaR has led to the results illustrated in the figure (Graph 1) above. It presents the calculated VaR for different annual portfolios with a 95% confidence level. The values

on the y-axis depict the highest daily financial setbacks (in millions TND) related to a 100 million TND public debt portfolio during the research period spanning from 2004 to 2019. Thus, over this period, a representative portfolio of 100 million TND could experience daily losses ranging from 0.2 to 0.46 million TND with a 95% confidence level. This result indicates reasonable control and management of the risk linked to the Tunisian external public debt portfolio. However, despite some relative stability in annual VaR over time, there are still recorded peaks, especially in 2013 (0.37264), 2016 (0.4523), and 2017 (0.463106899). These fluctuations are mainly due to the appreciation of one or all of the borrowing currencies relative to the TND.

Afterwards, a comprehensive analysis was conducted, delving into the currency-specific distribution of exchange rate risk about Tunisia's external public debt portfolio. The thetas, which reflect the level of risk associated with each currency in this portfolio, have been summarised in Table 3 (see the Appendix) and graphically represented below.

Graph 2

The calculation of Thetas by currency



Source: Author's calculation based on matrix computation results.

The observation of annual portfolios consistently (Graph 2 above) shows that the Japanese Yen has consistently presented the highest theta among the different currencies. This finding aligns with the previous conclusions of (Ajili, 2008) and (Omrane, 2012), who had

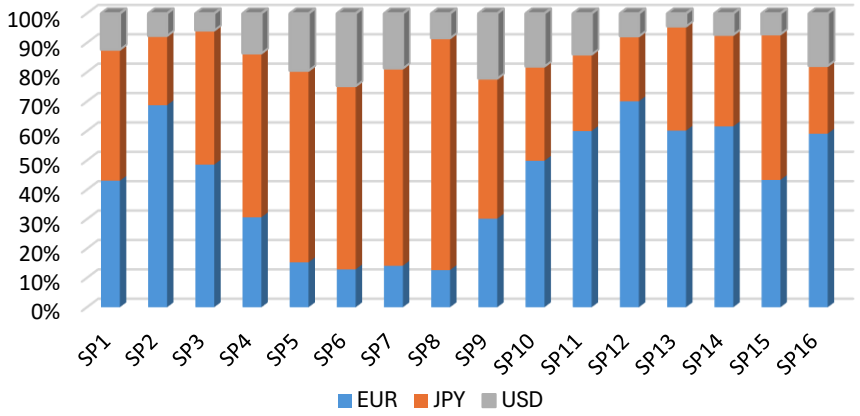
already identified the Japanese Yen as the currency with the highest level of exposure within Tunisia's external public debt.

During the period from 2004 to 2006, the US Dollar stood out as the currency with the lowest risk level, as evidenced by the minimum theta indicated in Table 4 (in the appendix). However, there was a change in the trend from 2007 to 2013, with the Euro surpassing the US Dollar in terms of stability. During this timeframe, the Euro emerged as the currency with the lowest risk in the portfolio.

During a third period, from 2014 until the end of the study in 2019, the US Dollar regained its position as the least exposed currency for Tunisia, followed by the Euro.

Graph 3

The decomposition of VaR by currency



Source: Author's calculation based on matrix computation results.

Once again, the detailed analysis of the VaR decomposition (as presented in Table 5 in the Appendix and illustrated in Graph 3 above) confirms the previous conclusions. It is established that the Japanese Yen contributes on average to around 44.45% "of the total VaR for Tunisia's external public debt portfolio. Meanwhile, the Euro shows an average contribution of about 42.51%, a proportion very close to that of the Japanese Yen (the currency deemed the riskiest within the portfolio). In contrast, the average contribution of the US Dollar is limited to 13%, marking the lowest share in this portfolio.

Consequently, the US Dollar can be utilised as a safeguard against currency exchange rate fluctuations in Tunisia's foreign public

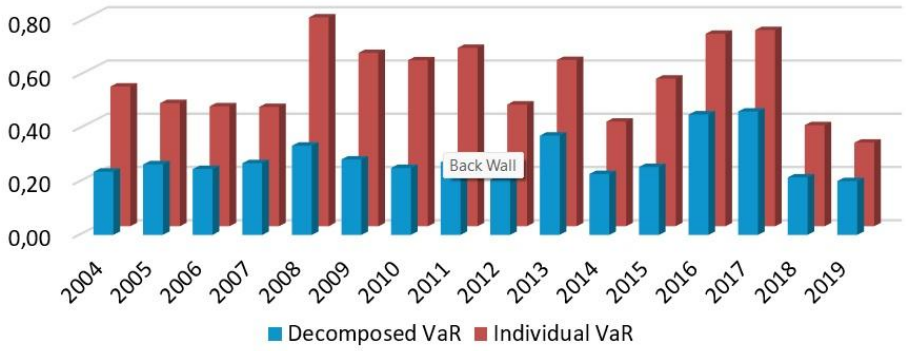
debt portfolio. Indeed, increasing its portion within the portfolio could lead to a reduction in the associated risk. Conversely, increasing the relative portion of the Euro or the Japanese Yen could amplify the comprehensive risk associated with this portfolio. However, it is essential to note that the negative impact of the Japanese Yen is more pronounced than that of the Euro.

The in-depth examination of the thetas related to different currencies, along with the detailed analysis of the decomposed VaR, converges to the recommendation of favouring an increase in the portion of Tunisian external public debt denominated in the US Dollar, at the expense of debts in Euro and especially Japanese Yen. This approach aims to optimise the management of the currency exchange risk inherent in the public debt portfolio.

However, it should be noted that this recommendation is made solely based on the financial analysis resulting from the VaR method. Therefore, it is crucial to consider, in subsequent studies, the relative shares of trade between Tunisia and the regions of Europe, Japan, and the United States (see Table 6 in the Appendix). These economic factors will play a key role in the optimal adjustment of currency allocations within the public debt portfolio.

Graph 4

The degree of VaR diversification



Source: Author's calculation based on matrix computation results.

The individual (non-diversified) VaR (as presented in Graph 4, above) which is the total of individual VaRs, comprises an average of 55% of the VaR for Tunisia's external public debt portfolio. Table 7 (in the Appendix) provides an overview of the split between non-diversified

VaR (the sum of individual VaRs) and diversified VaR (representing the risk associated with the interplay between the EUR, USD, and JPY). The above figure illustrates the level of VaR diversification (both individual and diversified) compared to the total VaR.

It is important to note that over the total period, individual VaR contributes on average 55% to the overall VaR, while diversified VaR represents only an average of 28.48% of the total VaR. This observation reflects effective exchange rate policy management by the Central Bank of Tunisia, as well as prudent management of Tunisia's external public debt.

5. Conclusion and policy recommendations

Managing external public debt is a major concern for many countries, especially those in the developing world. Exposure to foreign exchange risk is one of the most significant challenges these nations face, as it can have an impact on their public finances.

In this study, the VaR methodology was employed to evaluate the currency exchange risk linked to Tunisia's external public debt portfolio.

We observed that VaR, although widely used in the financial sector, has significant limitations when applied to external public debt. Specifically, the normality of exchange rate returns, a fundamental assumption of the VaR method, is not supported in our analysis of Tunisian data.

Therefore, in response to the issue of non-normality observed in the overall portfolio, we chose to decompose the global portfolio annually to move closer to normalising the data.

The analysis of annual portfolios reveals a consistent trend where the Japanese Yen consistently exhibits the highest theta among the various currencies. This observation corroborates previous findings from studies conducted by Ajili (2008) and Omrane (2012), which also identified the Japanese Yen as the currency with the highest level of exposure to risk within the Tunisian external public debt.

An important observation that emerges from our analysis is the predominance of the Euro in Tunisia's external public debt portfolio. This predominance is interesting because, from 2014 to the end of the study period (2019), the Euro exhibits a higher associated Value at Risk than the US Dollar. This finding might seem counterintuitive, but it can be explained by the natural hedge that the Euro provides to

Tunisia in terms of export earnings. Tunisia conducts a significant portion (almost 100%) of its exports with Eurozone countries. Therefore, a substantial portion of its export earnings is denominated in Euros. This means that when the Tunisian Dinar depreciates against the Euro, Euro-denominated export earnings increase in terms of Tunisian Dinars, partially offsetting the negative impact of depreciation on the external debt service denominated in Euros. Thus, this natural hedge reduces the exchange rate risk associated with the Euro in Tunisia's public debt portfolio.

However, it is becoming increasingly advantageous for Tunisia to opt for more borrowing in USD rather than EUR, especially when its trade transactions in USD increase. Several factors motivate such a decision:

1. Lower exchange rate risk: Tunisia faces a lower VaR in USD compared to EUR, indicating a currency less exposed to exchange rate risks.
2. Increased USD export earnings: An increase in export revenues denominated in USD compared to EUR would encourage Tunisian authorities to reconsider the structure and composition of their external public debt. This could be made possible by, among other factors, potential integration of Tunisia into the BRICS group (Brazil, Russia, India, China, South Africa), which could encourage Tunisia's foreign trade with new economic partners to the detriment of its traditional European trade partners.
3. Diversification of target markets: Tunisia should actively seek markets outside the Eurozone, particularly with countries in Asia, South America, or North America, which have strong trade ties with the United States. This reconfiguration of the external public debt portfolio toward the US Dollar can be considered a prudent strategy to manage exchange rate risks and leverage emerging trade opportunities.
4. Financial cooperation with other countries: Tunisia has an interest in expanding its financial cooperation with specific countries, including Algeria (100% of its debt to Tunisia is denominated in USD), Libya (100% of debt to Tunisia in USD), Saudi Arabia (with recent loan signed in USD 450 million in 2023), and others.

These strategies can help Tunisia better manage its exchange rate risks and capitalise on new commercial opportunities while maintaining prudent financial stability.

In conclusion, Value at Risk, despite its limitations, can provide valuable insights. However, it should be complemented by other measures and a thorough analysis of the specific national context. Therefore, Tunisian authorities and debt managers should continue to develop debt management strategies that consider these specificities and aim to minimise vulnerabilities to exchange rate fluctuations to ensure financial stability and long-term sustainability of the Tunisian economy.

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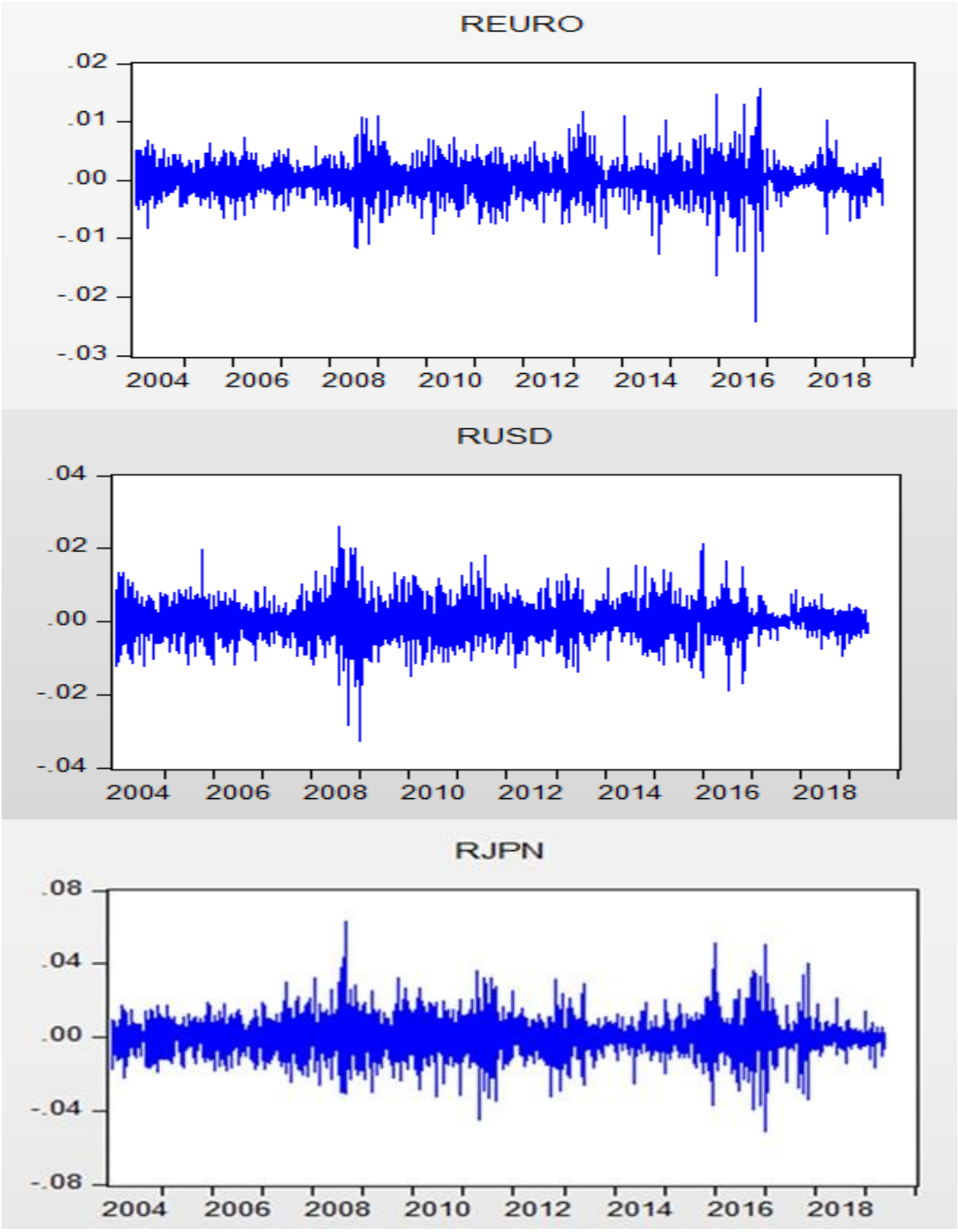
APPENDIX

Figure 1

Graphical Representation of Exchange Rate Returns



Source: Output from the EViews software.

Table 1

Descriptive Statistics of the Three Exchange Rates (02/01//2004-31/12/2019)

	EUR	JPY	USD
Mean	2.137888	16.57311	1.721938
Median	1.978670	16.30515	1.503875
Maximum	3.471050	28.12980	3.073120
Minimum	1.513900	10.31800	1.147400
Std. Dev.	0.504940	4.647974	0.529586
Skewness	1.103063	0.618112	1.076687
Kurtosis	3.288956	2.589787	2.935087
Jarque-Bera	827.1444	283.4611	775.4723
Probability	0.000000	0.000000	0.000000
Sum	8572.931	66458.19	6904.972
Sum Sq. Dev.	1022.152	86609.10	1124.369
Observations	4010	4010	4010

Source: Output from the EViews software.

Table 1a

Correlation matrix between the three exchange rates

	EUR	JPY	USD
EUR	1	0.922829	0.967294
JPY	0.922829	1	0.908068
USD	0.967294	0.908068	1

Source: Output from the EViews software.

Table 1b

Variance-covariance matrix

Covariance Analysis: Ordinary
Date: 05/31/23 Time: 11:59
Sample: 1/02/2004 5/16/2019
Included observations: 4010
Balanced sample (listwise missing value deletion)

Covariance Probability	EUR	JPY	USD
EUR	0.254901 -----		
JPY	2.165291 0.0000	21.59828 -----	
USD	0.258599 0.0000	2.234654 0.0000	0.280391 -----

Source: Output from the EViews software.

Table 2

Descriptive statistics of returns over the entire period

	REUR	RJPY	RUSD
Mean	0.000181	0.000206	0.000209
Median	0.000208	0.000104	0.000151
Maximum	0.015575	0.063230	0.026035
Minimum	-0.024284	-0.051041	-0.032776
Std. Dev.	0.002523	0.008535	0.004269
Skewness	-0.130748	0.203591	0.083125
Kurtosis	8.806952	7.493417	6.440863
Jarque-Bera	5644.182	3400.399	1982.313
Probability	0.000000	0.000000	0.000000
Sum	0.725172	0.827727	0.839031
Sum Sq. Dev.	0.025514	0.291944	0.073055
Observations	4009	4009	4009

Source: Output from the EViews software.

Table 3
Daily VaR (in millions TND)

Sub-period		Daily VaR
SP1	2004	0.2373293
SP2	2005	0.265051
SP3	2006	0.24758756
SP4	2007	0.269397
SP5	2008	0.3349582
SP6	2009	0.283242
SP7	2010	0.2512198
SP8	2011	0.2719759
SP9	2012	0.20758
SP10	2013	0.37264
SP11	2014	0.228727567
SP12	2015	0.2551319
SP13	2016	0.4523
SP14	2017	0.463106899
SP15	2018	0.21588469
SP16	2019	0.202095

Source: Author's calculation based on matrix computation results.

Table 4
Calculation of Theta (Ɵ) for each sub-portfolio

	REUR	RJPY	RUSD
2004	0.00743	0.03189	0.00646
2005	0.01186	0.01689	0.004
2006	0.00837	0.03265	0.00303
2007	0.00528	0.03988	0.00713
2008	0.00264	0.04662	0.01024
2009	0.00222	0.04458	0.01299
2010	0.00244	0.04801	0.00982
2011	0.00218	0.05641	0.00441
2012	0.00519	0.03411	0.01164
2013	0.0086	0.02293	0.00949
2014	0.01034	0.01865	0.00733
2015	0.01209	0.01585	0.00406
2016	0.01037	0.02536	0.0023
2017	0.01061	0.02231	0.0038
2018	0.00747	0.03543	0.00371
2019	0.01019	0.01649	0.00938

Source: Author's calculation based on matrix computation results.

Table 5
VaR Breakdown by Currency

	REUR	RJPY	RUSD	Total
2004	43.07%	44.65%	12.28%	100%
2005	68.77%	23.64%	7.59%	100%
2006	48.55%	45.71%	5.74%	100%
2007	30.62%	55.83%	13.55%	100%
2008	15.29%	65.26%	19.45%	100%
2009	12.90%	62.42%	24.68%	100%
2010	14.13%	67.21%	18.66%	100%
2011	12.65%	78.98%	8.37%	100%
2012	30.13%	47.76%	22.11%	100%
2013	49.87%	32.10%	18.03%	100%
2014	59.97%	26.11%	13.92%	100%
2015	70.10%	22.19%	7.71%	100%
2016	60.12%	35.51%	4.37%	100%
2017	61.56%	31.23%	7.21%	100%
2018	43.35%	49.60%	7.05%	100%
2019	59.08%	23.09%	17.83%	100%

Source: Author's calculation based on matrix computation results.

Table 6
Geographical Distribution of Tunisia's External Trade (Exports) in %

Sub-period	Japan	USA	EU
SP1	0.28%	1.45%	98.27%
SP2	0.27%	0.80%	98.93%
SP3	0.30%	1.73%	97.97%
SP4	0.33%	1.44%	98.23%
SP5	0.55%	2.41%	97.04%
SP6	0.53%	1.93%	97.54%
SP7	0.51%	3.34%	96.15%
SP8	0.79%	2.05%	97.16%
SP9	0.74%	2.69%	96.57%
SP10	0.61%	3.32%	96.07%
SP11	0.36%	2.03%	97.62%
SP12	0.25%	3.39%	96.35%
SP13	0.32%	2.44%	97.24%
SP14	0.16%	2.99%	96.85%
SP15	0.11%	3.68%	96.22%
SP16	0.20%	2.5%	97.29%

Source: National Institute of Statistics

Table 7

Comparison between Decomposed VaR and Individual VaR

	Decomposed VaR	Individual VaR	Diversification degree	%
2004	0.2373293	0.524797	0.287468	28.75%
2005	0.265051	0.462507	0.197456	19.75%
2006	0.24758756	0.450391	0.20280344	20.28%
2007	0.269397	0.448043	0.178646	17.86%
2008	0.3349582	0.784321	0.4493628	44.94%
2009	0.283242	0.650557	0.367315	36.73%
2010	0.2512198	0.623238	0.3720182	37.20%
2011	0.2719759	0.669997	0.3980211	39.80%
2012	0.207	0.4571	0.2495	24.95%
2013	0.37264	0.624263	0.251623	25.16%
2014	0.228727	0.392926	0.164198	16.42%
2015	0.2551319	0.554451	0.2993191	29.93%
2016	0.4523	0.722547	0.270247	27.02%
2017	0.4631068	0.736588	0.2734811	27.34%
2018	0.21588469	0.379691	0.1638	16.38%
2019	0.202095	0.3141	0.112	11.20%

Source: Author's calculation based on matrix computation results.