THE SHADOW DEFAULT-FREE REAL RATE OF RETURN

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Abstract

This paper attempts to resolve several intriguing observations. First, there is an unusually high correlation between the inflation-protected and nominal interest rates when the changes in the nominal interest rates are predominantly driven by inflationary pressure. This observation appears to contradict the Fisherian paradigm and the neo-Keynesian reasoning regarding the neutrality of monetary policy and real interest rates. Stiglitz (1983) argued that in the absence of intergenerational distribution effects, public financial policy has neither real nor financial effects on the optimal mixture of nominal and inflation-protected instruments. Second, a modest increase in the Break-Even Rate (BER) relative to a recent rise in expected inflation implies a lower Inflation Risk Premium (IRP) in a period with higher uncertainty about inflation. Third, the non-monotonic pattern of the volatilities of the default-free interest rates, where the volatility declines over maturity for the long-duration bonds. These observations may cast doubt on whether the default-free inflation-protected bond yields represent the real rates. This paper proposes to define a new concept of the shadow default-free real rate of return, derived directly from the market's equity and debt yields. The paper tests and confirms the hypothesis that this shadow rate of return is uncorrelated with the default-free nominal interest rates, highly correlated with changes in real GDP, and exhibits a monotonic volatility structure over maturity.

Keywords: inflation risk premium, forward premium, inflation-protected rate of interest

JEL Classification: G12, E43

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1. Introduction

A recent sharp rise in the nominal interest rates in most Western economies[1] has predominantly stemmed from inflationary pressures, which naturally have driven the market consensus of expected inflation upward[2]. During the same period, the Break-Even Interest rate (BEI) between the nominal and real interest rates has changed only moderately[3]. This observation explains why the correlation between these rates has increased significantly[4] i.e., the real rate consistently follows the nominal rate. In other words, a relatively constant BEI over this period, implying a high correlation between the nominal and inflation-protected rates, leads to the conclusion that the rise in expected inflation should have been fully offset by an equivalent decline in Inflation Risk Premium (IRP). However, this conclusion about the IRP cannot be verified empirically, as most likely both inflation expectations and the IRP have risen lately.

The above intriguing observation, clearly demonstrated in Section 2, requires a different explanation, inconsistent with mainstream paradigms such as Fisher's (1930). The Fisherian paradigm attributes the movements of the nominal rate, relative to the movements of the real rate, to changes in expected inflation. Under Fisher's paradigm, changes in the nominal rate that stem from inflationary pressures and are followed by similar changes in the real rate seem counterintuitive. Naturally, real economic factors can affect both rates, however, it is unlikely that throughout the entire recent period, the sole set of relevant factors was a real set of factors, especially in the past few years when monetary factors underlay the changes in the nominal rate. Furthermore, one can argue that inflationary pressures have an indirect, distorting real effect, but it is unlikely that these real effects are equally relevant for the nominal and

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[1] From a worldwide average of 1.9% in 2020 to 7.9% in 2023 (source: World Bank data).

[2] As shown consistently by the survey of inflation expectations by the University of Michigan.

[3] From mid-2022 to the end of 2024, the default-free BEI for 20-year maturity had almost the same value of around 2.6%.

[4] The correlation between the US default-free 10-year nominal and inflation-protected bond rates increased from 0.73 in 2020 to 0.88 in 2023. See Figures 1 and 2.

real rates, as is implied by the observed high correlation between the inflation-protected and nominal rates.

Moreover, monetary policy may explain the high correlation between the very short-term rates. However, it is hardly an acceptable explanation for the 10-, 20-, and 30-year rates, which also exhibit a high correlation.

Kraizberg (2025) establishes, theoretically and empirically, a relationship between the correlation of inflation-protected and nominal interest rates and the biases in the corresponding forward markets. As in Fama (1984), these biases measure the consistent difference between the forward and the ex-post realized prices. In this case, the causality is unclear, as one cannot assert that the forward biases affect the correlation or vice versa.

In contrast to the recently observed high correlation between the real and the nominal interest rates, many scholars such as Rogoff, Rossi, and Schmelzing (2022), Obstfeld (2023), and Blanchard (2022) have concluded, based on data before the 2022-4 sharp rise in inflation-protected government bond yields, that real interest rates have declined since the 1980s. Rogoff et al. (2022) argue that the downturn trend started as early as the 16th century, thereby concluding that "the sharp drop in the real interest rate in the 21st century, particularly in the years after the global financial crisis, has been arguably the most important macroeconomic development in modern times". They conclude that there is no compelling evidence of a break in the long-run trend in global real interest rates. The common arguments are that the main underlying factors pushing the rate down are demographic shifts, lower productivity growth, corporate market power, and safe asset demand relative to supply. Potential additional factors that can explain a downturn in interest rates in the past 400 years are technological improvements in information accessibility and improved efficient enforcement of recovery mechanisms that have led to increasing trust in commercial dealings. A similar view is expressed in Miller et al. (2024), who argue that the default risk premium has decreased. Obstfeld predicts that the economic factors "do not appear poised to reverse strongly enough to drive a big and durable rise in global real interest rates over the coming years". We probably need to distinguish between the long-term trend, which is strongly affected by better accessibility to information and more efficient enforcement, and the short-term trend, which seems to follow the nominal rate closely.

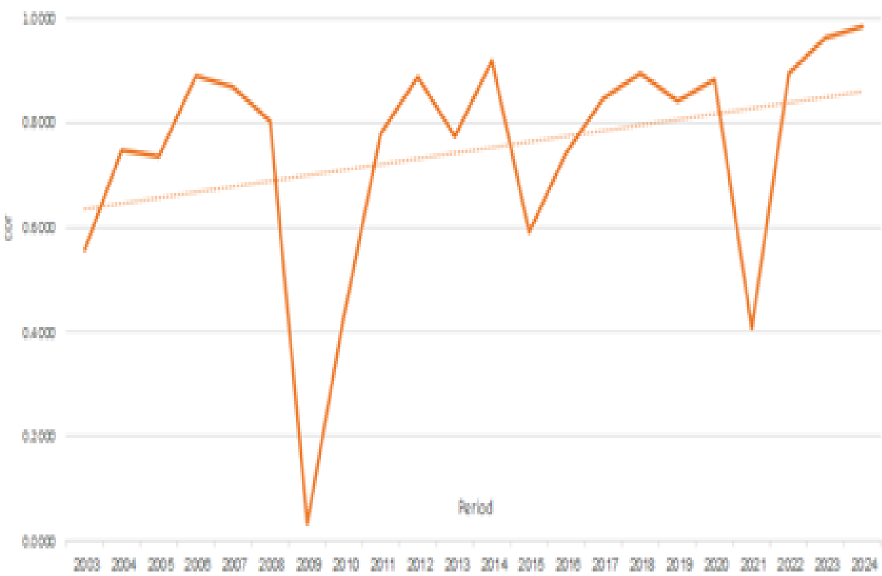
Blanchard (2023) advocates for Summers' (2014) concept of secular stagnation, noting that the important factor, r-g, (real rate of interest less economic growth rate), has been and still is negative. However, the rate of interest is one side of the equation, and perhaps we should compare the real rate of growth with the shadow real rate of return, which will be established and advocated below.

Given the declining trend in real interest rates worldwide, monetary policy decision-makers, perhaps following Wicksell's (1898) and Woodford's (2003) framework, have adopted the belief that real interest rates may not be high enough to curb the recent global upsurge in inflation. Underlying this policy decision is Wicksell's belief that the causality goes from interest rates to inflation, while Fisher's (1930) framework implies that the causality goes from inflation to interest rates. In other words, in Fisher's framework, inflation tends to affect the nominal interest rate, leaving the real rate unchanged, while in Wicksell's framework, a change in interest rates negatively affects the change in the inflation rate. Economists, using the term "long-term neutrality", argue that if Fisher's and Wicksell's effects have the same strength, the long-run real interest rate will be unaffected (for a discussion, see Anari and Kolary, 2016).

The observed high correlation between the default-free real and nominal interest rates is therefore intriguing, but no less surprising is the high correlation between the default-free inflation-protected interest rates and inflation. To the extent that the volatility in the nominal interest rates predominantly stems from shocks in expected inflation, the real interest rates should hardly follow the nominal interest rate. Moreover, a simultaneous rise in inflation and the nominal interest rate reduces the demand for real money balances, which in turn increases savings via a wealth effect, so that the real interest rate must fall in order to restore the equilibrium in the goods market. Thus, under both frameworks, Fisher's and Wicksell's, a very low or even negative correlation between the real and nominal interest rates is expected. In the short run, however, as suggested by Mundell (1963) and Tobin (1965), inflation shocks should affect the real interest rate, triggering some positive correlation between the real and nominal rates. Mishkin (1990), while arguing that Fisher's effect lacks robustness, finds that the correlation of estimated real rates and nominal interest rates is low in the 1931–1952 sample period and is even negative in the postwar period, corresponding to a negative correlation between the real rate and expected inflation. Plakandaras et al. (2023), using a 700-year data set, demonstrate the existence of a one-to-one Fisherian relationship between inflation and nominal interest rate co-movements and find that there is no impact on long-term real interest rates. The recent 2022–23 upsurge of inflation that led to a sharp increase in the nominal rates may lend itself to the hypothesis that the nominal and real interest rates should exhibit a more orthogonal relationship, but the data suggest otherwise. Figure 1 presents the behaviour of the correlation between the yields of the real and nominal interest rates. Figure 2 presents the behaviour of the correlation between the daily holding of real and nominal returns. Holding returns include daily accrued interest, accrued inflation, and capital gain/loss (see Appendix for the methodology). Both graphs demonstrate that in the recent period, there was a sharp increase in correlation between the real and nominal interest rates, reaching a level of about 70% for holding returns (Figure 2) or 95% for the yields (Figure 1).

Figure 1

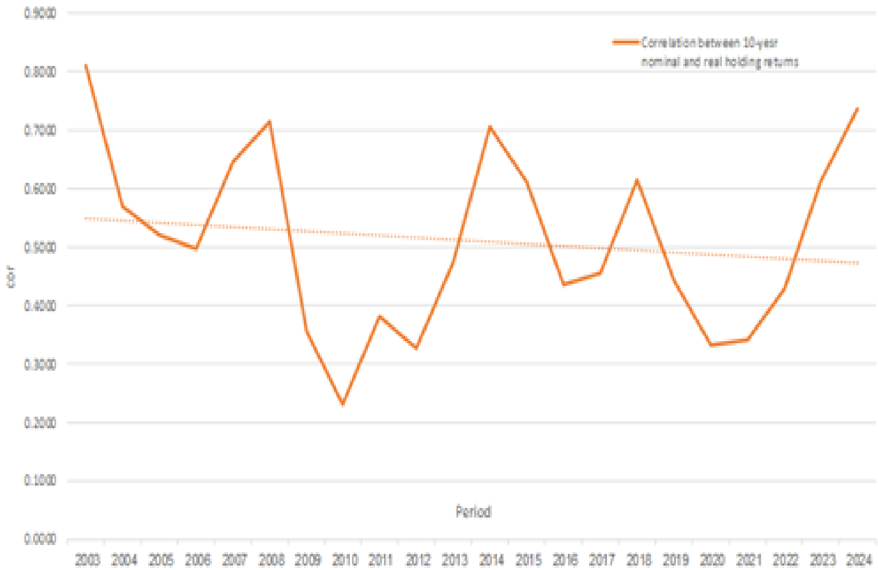
The correlation between nominal and real interest rates (U.S. Treasuries, 2003-2024)



Source: Based on daily yields extracted from FRED St. Louis, https://fred.stlouisfed.org/categories/22

Figure 2

The correlation between nominal and real holding returns (U.S. Treasuries, 2003-2024)



Source: Based on daily returns, extracted from FRED https://fred.stlouisfed.org/categories/22)

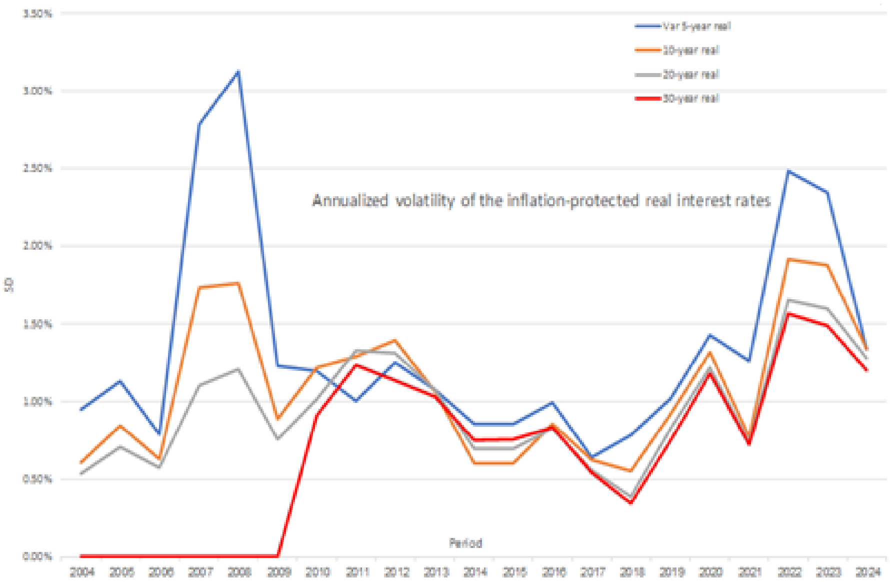
The second intriguing observation is the behaviour of the annualised[5] volatilities of the daily changes in interest rates. While the variability in the nominal rates is “normal” in the Keynesian sense of increasing with time to maturity and being fairly monotonic, the variability in the inflation-protected rates decreases with time to maturity and is non-monotonic during some periods.

Figure 3 depicts the variability in the inflation-protected interest rates.

[5] *Based on the assumption of random walk.*

Figure 3

Volatility of inflation-protected interest rates (U.S. Treasuries, 2004-2024)



Source: Calculated on the changes of daily yield in FRED, St. Louis

On the other hand, a monotonic pattern over time to maturity is obtained when calculating the difference (Phi) between the standard deviation of the changes in the nominal interest rate and the combined standard deviation of the changes in the real interest rate and the inflation rate. Presumably, this difference is highly correlated with the so-called IRP (inflation risk premium), which is monotonically increasing over maturity.

Table 1 shows the average Phi, and Figure 4 presents the behaviour of Phi for the past 20 years.

Table 1

Phi - A measure of a biased standard deviation

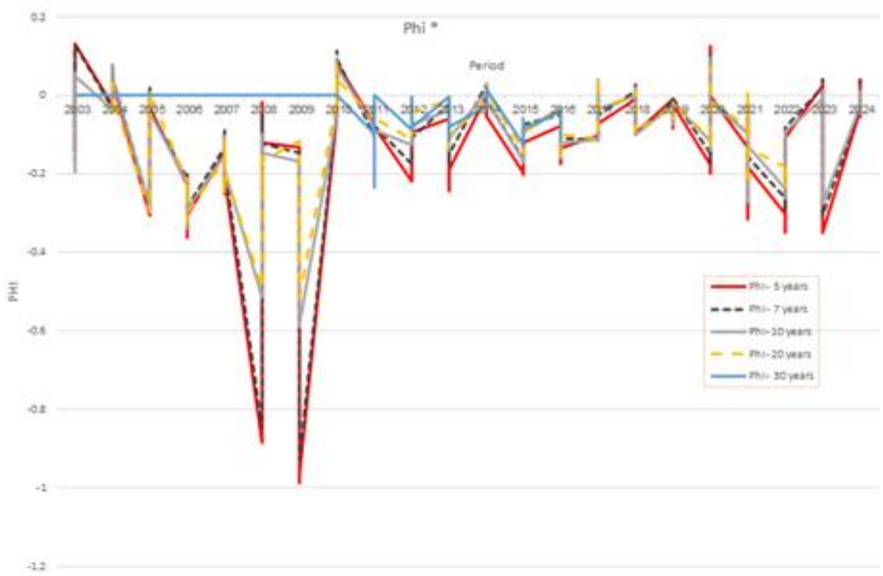
Maturity (years)	Phi
5	-1.64880
7	-1.03572
10	-0.55644
20	-0.13354
30	-0.11219

Note: Phi, the difference between the standard deviation of the nominal interest rate and the combined standard deviation of the real interest rate and inflation (percents). $Phi = SD[(i_{t+1} - i_t)] - SD[(r_{t+1} - r_t) + (\pi_{t+1} - \pi_t) + (r_{t+1}\pi_{t+1} - r_t\pi_t)]$, *where* i_t, r_t *are the nominal and real interest rates, respectively, at time t and* π_t *is the CPI/u level, seasonally unadjusted.*

Source: Based on data extracted from FRED.

Figure 4

The difference between the standard deviation of the nominal interest rate and the combined standard deviation of the real interest rate and inflation (U.S. Treasuries, 2003-2024)



Sources: Based on data extracted from FRED.

Standard models in the literature, such as affine term-structure models, that attempt to decompose the nominal yields into real, expected inflation, and a risk premium, would conclude that it is not trivial to identify parameters that may substantiate inconsistent variability trends that are observed in nominal and inflation-protected bond data.

The above intriguing observations may prompt the question of whether there is a common denominator that may explain or link them together. We believe that there is an explanation rooted in the definitions of the default-free real rate of return. We wish to construct an alternative measure of the real rate of return, labelled here as the shadow default-free real rate of return. This real rate of return is a specific linear combination of the real rate of interest and the expected real equity rate of return. If the real interest rate has historically declined, as argued in the literature mentioned above, and given a relatively stable expected equity rate of return, then the shadow real rate of return must have decreased as well; however, this conclusion may not be substantiated empirically, at least not in the past twenty years.

Some of the intriguing observations may be explained by utilising the shadow real rate of return, as will be demonstrated in this paper's empirical results. The shadow default-free real rate of return should not be confused with any commonly used definition. In the past century, various definitions of the 'real rate' have been proposed or identified as follows:

1. The Fisherian hypothesis implies that the real interest rate is the difference between the nominal rate of interest and expected inflation. Marshal (1890) suggested that the real rate is the above difference, less an additional term, which is the arithmetic product of the nominal and real rates. Subsequently, the definition was modified to include an inflation risk premium (IRP), being the covariance of the marginal rate of substitution of consumption over time and inflation (See earlier studies in Cox, Ingersoll, and Ross (1985), Benninga and Protopapadakis (1991), and more recent work. Mishkin (1990) investigates the behavior of the real rate and inflation and finds that "the real rate, whether adjusted or unadjusted for taxes, is negatively correlated with inflation".

2. Economists cite Wicksell's (1890) natural/neutral rate as the equilibrium real interest rate, consistent with the eventual full capacity of the utilisation of available resources in the context of low and stable

inflation. Wicksell (1998) wrote: “There is a certain rate of interest on loans which is neutral in respect to commodity prices and tends neither to raise nor to lower them”. It is an unobservable interest rate on loans that is neutral concerning commodity prices and tends neither to raise nor to lower them. Woodford (2003), Andrés, López-Salido, and Nelson (2008), Barsky, Justiniano, and Melosi (2014), Cúrdia, Ferrero, and Ng, G.C., Tambalotti (2015), and Holston, Laubach, and Williams (2017) incorporate the natural interest rate concept into new Keynesian models. Laubach and Williams (2003, 2016) operationalise this notion by defining the natural rate as the real short-term interest rate consistent with the economy operating at its full potential once transitory shocks to aggregate supply or demand have abated. Lopez-Salido et al. (2020) incorporate information from long-run survey forecasts of inflation, as opposed to market data, and demonstrate a sharper decline in the estimated natural interest rate since the early 2000s. Carr (2009, 2018) combines novel finance models, starting with Vasicek (1977) and the notion of the natural interest rate, by introducing factors other than time, demonstrating that bond market expectations of riskless rates converge to the natural rate of interest. Obstfeld (2023) distinguishes between the Natural and Neutral rates: "By natural rate, I will mean the real rate of interest prevailing in an equilibrium where price rigidities are absent. By neutral rate, I will mean the real policy rate of interest that eliminates inflationary or deflationary pressures".

3. The yield to maturity on default-free, inflation-protected government bonds (TIPS), observable as market data since 1997. The extent to which these rates represent the real rate of interest is questionable, considering several empirical and theoretical observations:

- Liquidity risk. The TIPS market, in which inflation-protected bonds are traded, is relatively new and has attracted less attention than the traditional nominal treasuries. Liquidity risk may arise from multiple market frictions, such as limited investor participation, transaction costs, the composition of market participants, funding constraints, and net supply imbalances between the two types of securities

- Deflation-protected provision of the US and Japanese TIPS. The overall inflation component accrued to the value of the TIPS may not be negative, as stated by the terms of the bonds, thereby creating an option for the bondholders. (see Grishchenko, Vanden, and Zhang, 2011; Kitsul and Wright, 2012; and Hiraki and Hirata, 2020, who study the Japanese inflation-linked bonds, in which a principal protection feature prevailed after 2013).

- Adapting the Fisherian causality[6]. If the default-free nominal interest rate fluctuates in response to changes in expected inflation, the default-free, inflation-protected interest rate should not follow suit. Yet, the default-free nominal and inflation-protected rates of interest are highly correlated, at least in the past 20 years (see Kraizberg, 2025, who argues that the high correlation can be partially explained by corresponding forward premiums). Mishkin (1990), before the first issuance of the TIPS while using the difference between the nominal interest rate and expected inflation as a proxy for the real interest rate, finds that the correlation between the real rate and nominal interest rates is low in the 1931-52 sample period and is even negative in the postwar period.

The shadow[7] default-free real rate of return, established here, unlike the natural real interest rate, may be extrapolated from the financial markets data. Specifically, it is a measure extrapolated from the private defaultable sector, adjusted for the default-free rate of return, while avoiding the need to estimate the so-called "government equity share"[8] in the economy. The inflation-protected interest rates constitute just one component of the equation when deriving the shadow real rate of return, where the latter is a linear combination of the inflation-protected interest rates and an extrapolated default-free real rate of return of equity. Scaling market participants along their risk preferences, the government is considered the most risk-tolerant, less than fully diversified, relative to the less risk-tolerant, fully diversified private sector. In a perfect economy, one could argue that the government should hold equity financed by debt, while the less risk-tolerant private sector would be a net lender. Under this framework, if the government had been a risk-tolerant agent with a major equity position, its borrowing rate, i.e., the inflation-protected rates, would reflect the true real interest rate (see a different relevant view in Hall

[6] *Alternatively, under a particular case in which Fisher and Wicksell's effects are of the same strength, (iii) holds as well.*

[7] *A somewhat unrelated strand of literature refers to shadow rates as the potential negative nominal rates of interest in a Zero Lower Bound economy. See a survey in Lee (2020).*

[8] *The so-called government equity position is not a marketable security and can only be estimated through government spending as a portion of GDP.*

(2017). However, this is hardly the case in Western economies, and therefore, using the inflation-protected rates as a benchmark for the real rate may be misleading. The extrapolated shadow real rates of return, if estimated correctly, may provide a better picture as is demonstrated in the empirical part of this paper.

This paper wishes to empirically confirm the concept of a shadow default-free real rate of return, implied from the financial markets, which is neither the default-free inflation-protected rate of interest nor the natural real rate of interest. We hypothesise and test several implications:

i. We expect to find a much lower correlation between the shadow default-free real rate of return and the default-free nominal rate of interest when the changes in the nominal rates predominantly stem from inflationary pressures. If this assertion is confirmed, it contrasts the high observed correlation between the default-free inflation-protected and nominal interest rates. This finding, if confirmed, would be supporting evidence for the Fisherian hypothesis.

ii. Similarly, we expect to find a very low correlation between the shadow real rate of return and inflation, unlike the puzzling high correlation between the inflation-protected interest rate and inflation.

iii. We may contrast the common belief that the real interest rate exhibits a long-term declining trend, demonstrating, at least in the past 20 years, a non-decreasing trend in the shadow real rates of return.

iv. Finally, we expect to find a monotonic increase in variability of the shadow real rates of return over maturity, as opposed to the non-monotonic pattern exhibited by the inflation-protected interest rates.

We will conduct further tests to determine whether the shadow real rate of return more accurately reflects changes in real output. Laubach and Williams (2003, 2015) argue that the recent decline in real interest rates, along with a decrease in the natural interest rate, is associated with a decline in potential GDP (see Figure 6 in LW). Likewise, we hypothesise that the shadow real rate of return is directly linked to the realised or expected rate of change in real output. If the shadow real rate of return derived from financial markets serves as a proxy for the overall, economy-wide real rate of return, we expect to observe a strong correlation between these two variables. Conversely, if we do not find a significant correlation, we may attribute this to misspecification or measurement errors in the empirical estimation of the shadow real rates. In that case, we will calibrate our findings by adjusting technical parameters. Rogoff, Rossi, and Schmelzing (2022,

Table 7) emphasise, through their literature survey using data from 1311 to 2021, a significant negative correlation between the real interest rate and the Baxter-King-filtered[9] long-run components of both aggregate population growth and aggregate real output growth. We will adopt these variables to test our hypotheses regarding the shadow real rate of return.

2. The setting

By definition,

$$\Delta\left(i_{t,t+1} - r_{t,t+1}\right) \equiv \Delta\left(BEI_{t,t+1}\right) \equiv \Delta\left(E_t\pi_{t,t+1} + IRP_{t,t+1}\right) \qquad \text{(D-1)}$$

where $i_{t,t+1}$ is the one-period, default-free nominal interest rate, and $r_{t,t+1}$ is the one-period default-free, inflation-protected interest rate, BEI is the break-even interest rate, and $E_t\pi_{t,t+1}$ is the expected one-period inflation rate given the information set known at time *t*. IRP, the Inflation Risk Premium, captures the difference between the BEI and expected inflation. As per our discussion in the introduction, if the BEI is stable, as it was in the past three years, then it must be true that,

$$\Delta\left(i_{t,t+1} - r_{t,t+1}\right) \equiv \Delta\left(BEI_{t,t+1}\right) \equiv \Delta\left(E_t\pi_{t,t+1} + IRP_{t,t+1}\right) = 0 \qquad \text{(D-2)}$$

(D-2) implies that an increase in expected inflation, as surely occurred since interest rates started to rise, then the IRP must have declined to offset the increasing expected inflation. However, as the risk of higher inflation rose, so did the IRP. While a constant BEI lends itself to a high correlation between the nominal and real interest rates, the invalidation of the identity (D-2) raises a question about the proper explanation for the high correlation between these rates.

We shall start with a basic argument. If the changes in the nominal interest rate are predominantly affected by changes in expected inflation, then it should be captured by the coefficient $\beta_{t,t+1}$ in the following equation:[10]

$$\Delta i_{t,t+1} = \alpha_{t,t+1} + \beta_{t,t+1}\,\Delta E_t\pi_{t,t+1} + \epsilon_{t,t+1} \qquad (1)$$

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[9] *Baxter and King (1999 remove short and medium-run fluctuations from all variables.*

[10] *In Fama (1975) and Mishkin (1990) the implied causality is reversed* $\pi_{t,t+2} - \pi_{t,t+1} = \alpha_{t+1,t+2} + \beta_{t+1,t+2}(i_{t,t+2} - i_{t,t+1}) + \epsilon_{t+1,t+2}$.

$$\Delta i_{t,t+1} = i_{t+1,t+2} - i_{t,t+1} \tag{1a}$$

$$\Delta E_t \pi_{t,t+1} = E_t \pi_{t+1,t+2} - E_t \pi_{t,t+1} \tag{1b}$$

where $\pi_{t,t+1}$ is the realized one-period inflation rate from time *t* to *t+1*.

The relationship between $E_t \pi_{t,t+1}$, the expected one-period inflation rate at time *t*, and the realized inflation is,

$$\pi_{t,t+1} = E_t \pi_{t,t+1/\varphi_t} + \epsilon^{\pi}_{t,t+1} \tag{2}$$

where $\epsilon^{\pi}_{t,t+1}$ is the forecast error**.** If rational expectations hold, then, $E\epsilon^{\pi}_{t,t+1} = 0$. If $\epsilon^{\pi}_{t,t+1}$ consistently differs from zero it provides an ex-post estimate for the IRP, denoted here as $E\epsilon^{\pi*}_{t,t+1}$.

Then (1) can be written as:

$$\Delta i_{t,t+1} = \alpha_{t,t+1} + \beta_{t,t+1}\ \Delta\pi_{t,t+1} \quad + \Delta E\epsilon^{\pi*}_{t,t+1} \tag{3}$$

$\beta_{t,t+1} = 1$ implies the Fisherian view. If the concurrent $\beta_{t,t+1}$ differs from one, it may also indicate a delayed effect of inflation on the nominal interest rate. In contrast with the Fisherian view, the changes in inflation may be related to the corresponding real interest rate $r_{t+1,t+2}$ as long as $\beta_{t,t+1} \neq 1$.

By definition, the default-free inflation-protected interest rate, $r_{t,t+1}$, is

$$\begin{aligned} r_{t,t+1} &\equiv i_{t,t+1} - E_t\pi_{t,t+1} + E\epsilon^{\pi*}_{t,t+1} \\ &\equiv i_{t,t+1} - E_t\pi_{t,t+1} + IRP_{t,t+1} \end{aligned} \tag{4}$$

Then, from (1-3) it follows that

$$\Delta r_{t,t+1} = \alpha^*_{t+1,t+2} + \beta^*_{t,t+1}\ \Delta\pi_{t,t+1} + \Delta E\epsilon^{\pi*}_{t,t+1} \tag{5}$$

where $\alpha^*_{t,t+1} = -\alpha_{t,t+1}$ and $\beta^*_{t,t+1} = 1 - \beta_{t,t+1}$.[11]
$\beta_{t,t+1} = 1$ implies $\beta^*_{t,t+1} = 0$, i.e., the inflation-protected rate is orthogonal to the expected inflation, thereby orthogonal to the nominal rate.

Following Fama (1984) and Hardouvelis (1984), if rational expectations hold, then,

11 Inserting -Δt,t+1 on both sides of (1) and substituting for (2) and using the definition (3) and finally multiplying by -1.

$$\beta^*_{t,t+1} = \frac{\sigma^2(\Delta r_{t,t+1}) + \rho_{\Delta r_{t,t+1},\Delta\pi_{t,t+1}}\sigma(\Delta r_{t,t+1})\sigma(\Delta\pi_{t,t+1})}{\sigma^2(\Delta r_{t,t+1}) + \sigma^2(\Delta\pi_{t,t+1}) + 2\rho_{\Delta r_{t,t+1},\Delta\pi_{t,t+1}}\sigma(\Delta r_{t,t+1})\sigma(\Delta\pi_{t,t+1})} = \frac{\sigma^{*2} + \rho_{\Delta r_{t,t+1},\Delta\pi_{t,t+1}}\sigma^*}{1 + \sigma^{*2} + 2\rho_{\Delta r_{t,t+1},\Delta\pi_{t,t+1}}\sigma^*} \quad (6)$$

where σ^* is the ratio of the standard deviation of changes in the inflation-protected interest rate to the standard deviation of the changes in inflation. ρ is the correlation coefficient between the changes in inflation-protected interest rate and the changes in inflation. Interestingly, even if the real interest rate was orthogonal to the rate of inflation, $\beta^*_{t,t+1}$ could still be positive while the effect of the slope is offset by a negative constant.

In practice, $\beta_{t,t+1} \neq 1$. Both $\beta_{t,t+1}$ and $\beta^*_{t,t+1}$ are positive and smaller than one, which may explain the high correlation between the nominal and the inflation-protected interest rates. However, the observed values of these coefficients are the consequence of this correlation, rather than the cause.

We propose a measure of the *shadow real rate of return*, hypothesising that it will exhibit a low correlation with the nominal interest rate. The shadow real rate of return is a specific weighted average of the expected equity rate of return and the rate of marketable obligations. The methodology of estimation of the shadow real rate will be explained in detail below, but first, we wish to provide one possible explanation for the hypothesised low correlation between the shadow real rate and the nominal interest rate.

We hypothesise that the expected shadow real rate of return, $E_t r^s_{t,t+1}$ (as opposed to the inflation-protected interest rate $r_{t,t+1}$) is a linear combination of the expected overall market equity rate of return, $E_t x_{t,t+1} = ln\,(\frac{E_t X_{t+1}}{X_t})$ and the nominal interest rate on the obligation, $i^l_{t,t+1}$, with weights of ω and (1-ω), i.e.

$$E_t r^s_{t,t+1} = \omega E_t x_{t,t+1} + (1-\omega) i^l_{t,t+1} \quad (7)$$

If $i^l_{t,t+1}$ increases due to some inflationary pressure while the shadow real rate remains constant, the explanation may be related to the change in equity rate of return, or changes in the weights of ω and 1-ω, i.e.

$$d(E_t r^s_{t,t+1}) = \ d(\omega E_t x_{t,t+1}) + d((1-\omega) i^l_{t,t+1} = 0 \qquad (8)$$

Then, if $d(i^l_{t,t+1})\ is\ positive\ but\ \ d(E_t r^s_{t,t+1}) = 0$, it must be true that

$$d(i^l_{t,t+1}) = \frac{d\omega}{1-\omega\left[E_t x_{t,t+1} - i^l_{t,t+1}\right]} + \frac{\omega}{(1-\omega)d\left(E_t x_{t,t+1}\right)} \qquad (9)$$

3. Methodology

We will apply the above setting to derive the *default-free shadow real rate of return*. We proceed with a two-stage estimation procedure. The first stage of the procedure is comprised of the following sub-steps:

- Estimation of the market defaultable nominal rate of interest.
- Estimation of the market leverage ratio of defaultable instruments.
- Estimation of the market expected nominal rate of return on equity.
- Estimation of the market's overall expected nominal rate of return.
- Estimation of the default-free shadow real rate of return.

3.1. Data specifications

The following data sets were implemented in the empirical estimations:

- S&P index, daily, 4 pm prices. Source: FRED, St. Louis Fed 1998-2024.
- S&P Futures, daily 4 pm prices of the nearby contract, excluding the price at the contract expiration day, and instead the new nearby contract price is utilized. Source: Chicago Mercantile Exchange, CME, 1999-2024.
- Nominal treasury interest rates (YTM), daily at 3 pm, 1-30-year treasury bonds.
- Inflation-protected interest rates, daily 3 pm (YTM), for 5-30-year inflation-protected government bonds. Source: US Department of the Treasury 2003-2024, and FRED 2000-2003.
- The above U.S. Treasury bond yields represent the default-free interest yields. The daily yields are for the latest-issued existing treasury notes or bonds. Given the frequency of issuances of these instruments, the deviation from the actual maturity date is less than two weeks for maturity of up to 5 years, less than a month and a half for 10-year maturity, and less than 6 months for 20 and 30-year maturities. Similar deviations apply to the inflation-protected yields.

- S&P dividend rates. Ex-post quarterly declared, weighted average, dividend rates of the S&P firms.
- Nominal interest rates for S&P firms 2000-2024. Daily rates of Aaa and Baa-rated bonds for firms, which comprise the S&P index. Source: Moody's, 2000-2024, also in FRED.
- S&P firms' financial reports. Source NYSE.

3.2. Estimation of the market defaultable nominal rate of interest

The nominal rate of interest of the S&P firms is established, using the weighted average rating of the S&P firms, based on Moody's average ratings between Aaa and Baa (or S&P and Fitch equivalent ratings). The way it was calculated is as follows: We ordinally ranked Moody's ratings between Aaa (=1) and Baa3 and below (=11). Then, we sum the relative debt size over all firms, multiplied by rating. The outcome is a number between 1 and 11 multiplied by Moody's average interest rate for that category, which represents the average interest rate.[12]

The average maturity dates fluctuated between 8.9 and 10.2.[13]

3.3 Estimation of the defaultable market leverage ratio (quarterly)

Utilising the book values of the liabilities of all S&P firms may misrepresent the leverage ratio. Thus, we incorporate daily market data, quarterly book figures, and Merton's (1974) model, dividend-adjusted, to estimate the quarterly leverage weights in market terms. Specifically, the iterative process[14] includes the following variables:

- ✓ Daily equity value, as the total market capitalisation of the S&P index.
- ✓ Average duration of all rated S&P firms
- ✓ Nominal interest rates on government bonds with a maturity date close to the average duration of S&P debts. End of quarter.

[12] *Aaa[(wi*RATINGi)/11]+iBaa3[(1-wi)*RATINGi)/11], where wi is the debt size of an S&P firm relative to all S&P firms' debt in face value (quarterly) and. iAaa and iBaa3 are daily rates. Over the past 25 years [(wi*RATINGi)/11], it fluctuated moderately between 0.71 and 0.74 (about A3/A-). Firms that were Bloomberg, Goldman Sachs not rated by either Moody's, S&P, or Fitch are omitted.*

[13] *Source: S&P, Bloomberg. We did not use duration as a measure of the horizon. The difference is negligible for the short-term instruments.*

[14] *The overall asset value is iteratively implied from the market capitalisation plus the iteratively derived value of the liabilities, utilising Merton (1974), thereby setting the leverage in market terms.*

- ✓ Actual and declared dividend rate at the end of a quarter.
- ✓ The annualised variance of the assets' rates of return was implied from the variance of daily changes in the S&P index price.
- ✓ Total face values of the liabilities of all S&P firms at the end of each quarter, applied to the following quarter.

The two series, book or market leverages, produce very different results. The book leverage tends to elevate the real rate of return, but both series are highly correlated as the level of the rates of interest affects the value of the liabilities and equity in the same direction.

3.4. Estimation of the expected market equity nominal rate of return

Estimating the expected equity rate of return is central to scholarly research in finance. We wish to estimate the expected market equity nominal rate of return for the horizons of 7,10,20, and 30 years. We could rely on theoretical General or Partial Equilibrium frameworks such as the myopic CAPM, or the continuous time framework under Risk Neutral Valuation, but these estimates are as robust as the estimates of the underlying parameters. This approach will be implemented by the estimation of the implied market expectations embedded into future prices. Additionally, we use statistical prediction models that implicitly assume that the market forms expectations based on a current set of information, which is strongly affected by past observations. We realise that models may be robust in the short run in the sense that the predictions conform to the realised values, but given the central focus of this paper, we only wish to derive the market expectations rather than the prediction of the actual outcomes.

We adopted two commonly used approaches: time series forecasts (Box-Jenkins ARIMA) and implied projection from data comprised of futures prices. See Mishkin (1990) for a discussion of additional Econometric issues.

3.4.1 The use of Futures data to infer the expected equity rate of return

Fama (1984) raises doubts about the predictive power of the forward market prices concerning future spot prices. Forward prices reflect a concurrent no-arbitrage condition (CIP) and are consistently biased relative to realised future prices. Fama and Bliss (1987) and Campbell and

Shiller (1991) showed that the forward rates consistently deviate from the spot rates at a one-year horizon. However, they showed that forward-spot spreads do seem to predict changes in interest rates at longer horizons. Stambaugh (1988) and Cochrane and Piazzesi (2005) extend the finding of time-varying expected bond returns by forecasting returns with all available yields, not just single yields with specific maturities, finding substantially more return predictability.

If the CIP holds, the concurrent market rates of the forward and spot rates have little informative value, but the biased forward rate relative to the ex-post spot rate might. Following the discussion in the literature regarding the empirically demonstrated finding that the forward prices are biased estimators of the expected prices, we will extract the expected equity rate of return utilising the observed consistent biases.

In principle, if the absence of arbitrage opportunities is assumed, the futures prices of equity should follow:

$$F_{t,t+1} = X_t e^{(i_{t,t+1} - d_{t,t+1})\tau} \tag{10}$$

where $F_{t,t+1}$ is the current futures price for delivery at time *t+1*, t=(*t+1)*-*t*. X_t is the current equity price, $i_{t,t+1}$ is the nominal default-free interest rate for bonds mature at time *t+1*, and $d_{t,t+1}$ is the dividend rate on equity. $F_{t,t+1}$ reflects the default-free rate of interest and the dividend rate, but, in practice, it contains two valuable pieces of information.

First, if expectations are rational, and we observe a consistent, statistically significant difference between the futures prices and the ex-post equity price, i.e.

$$\frac{E_t X_{t+1}}{X_t} = -\alpha_t + (1 - \beta_t) F_{t,t+1}/X_t + \varepsilon_t = \frac{X_{t+1}}{X_t} \; /(E\varepsilon_t = 0) \tag{11}$$

where $E_t X_{t+1}$ is the expected equity value under rational expectations if $E\varepsilon_t = 0$. X_{t+1} is the realised price of equity at time *t+1*. α_t is a constant term and β_t, If different from one, it reflects the biased forward price given the realised price at time *t+1*. Thus, β_t , assuming that $E\varepsilon_t = 0$, may contain valuable information, trivially extracted, about expected equity yield.

Second, β_t reflects the information at time t, but *b* may change over time. Thus, the outcome $\beta_t - \beta_{t-1}$ is a valuable piece of information,

$$\frac{d\frac{E_t X_{t+1}}{X_t}}{dt} = d\alpha + (1-\beta_t)\frac{dF_{t,t+1}/X_t}{dt} - F_{t,t+1}/X_t\frac{d\beta}{dt},\ \text{given } E\varepsilon_t = 0,\ \text{and } dt \to 0 \quad (12)$$

For example, a change in *b*, ceteris paribus, implies a similar change in expected equity yield. See Söderlind and Svensson (1997) for a similar approach.

3.4.2 Time series forecasting

We will compare the random walk, Box-Jenkins' ARIMA (*0,1,0*) formulation with various *ARIMA* (*p,d,q*) models[15], while inserting, (i) the order of differencing *(d)* that needs to stationarize the behaviour of overall market equity price, or the order that minimises the standard deviation of the error term[16], (ii) setting the Box-Cox's lambda to zero, (iii) all statistically significant *MA* factor (*q*), (iv) including the mean, (v) the AR factor (*p*), and (vi), the Akaike (1985) Information Criteria (AIC) is minimised:

$$AIC = MIN\left[\sigma_e^2 + \frac{2(P+q)}{n}\right] \quad (13)$$

We apply the Augmented Dickey-Fuller test for non-stationarity with a cutoff significance level less than 0.05 for nonstationarity, thereby setting the order of differencing, d, as follows:

$$(X_t - X_{t-1})/Q_{t-1} = \alpha + \beta t + \theta_1(X_{t-1} - X_{t-2})/X_{t-2} \ldots .\theta_s(X_{t-s} - X_{t-s-1})/X_{t-s-1} + \cdots. \quad (14)$$

With d=2 and 5 lags, we reject the null hypothesis of nonstationarity as p>.05, as shown below:

ANOVA

Model		Sum of Squares	df	Mean Square	F	p
M_1	Regression	$5.900\times10^{+6}$	8	737446.332	0.596	0.782
	Residual	$9.637\times10^{+9}$	7790	$1.237\times10^{+6}$		
	Total	$9.643\times10^{+9}$	7798			

[15] *p, d, and q are non-negative integers, p is the order (number of time lags) of the autoregressive model, d is the degree of differencing (the number of times the data have had past values subtracted), and q is the order of the moving-average model.*
[16] *Most likely higher than one as we expect a time-varying trend.*

We also apply Ljung-Box's Q^* statistic to estimate the total number of lags and reject the hypothesis of randomness at 0.05:

$$Q^* = n(n+2)\sum_{l=1}^{h} \widehat{\rho_l^2}/(n-l) \qquad (15)$$

Where *n* is the sample size *(n*=7807 is our sample size, starting from 1/1/2003, including 1939 observations of the market equity prices, prior to 1/1/2003), ρ_l is the autocorrelation at lag *l*. The hypothesis is that, in the presence of ARCH components, at least one of the estimated $\alpha_i \alpha$coefficients in (10) must be significant,

$$e_t^2 = \delta_0 + \sum_{i=1}^{q} \delta_i e_{t-i}^2 \qquad (16)$$

We experimented with Box-Cox's lambda: -1.5, 1, 1.5, and the differences in the various outcomes were negligible.
The version ARIMA (*1,2,1*) with a constant μ_t , and $\theta,\ \delta,$ seems acceptable for our purpose. The final model is:

$$(EX^*{}_{t+1})^m = (1 + \mu_t + \theta_1(X^*{}_t - X^*{}_{t-1}) + e_t^2 + \delta_1\, e_{t-1}^2\,)^m \qquad (17)$$

Where: $X_{t-n}{}^* = (X_{t-n} - X_{t-n-1})/X_{t-n-1}$, $EX^*{}_{t+1}$ is the out-of-sample short-term forecast, and *m* is the annualising factor. The crucial assumption is that $(EX^*{}_{t+1})^m$ is time-invariant. That is, the market's expected long-term return at any point in time is constant over the horizon, and yet it may change over time. This is a reasonable assumption, as in practice, the market forms expectations for up to one year or so, and it is unlikely that at the same point in time, the expectations for, say, five years ahead, are very different.

3.5. Estimation of the market overall expected nominal weighted rate of return.

Given the weighted average interest rate on S&P firms' obligations (3.2), and the corresponding expected equity rate (3.4), both for the same average maturity of the obligations, [17] the overall expected nominal rate of return is established as a linear combination of the above rates, given the weights as in 3.3. Thus, the market's overall expected nominal rate of return, $i_{t,t+1}^*$ is given by,

[17] *Based on the average maturity of S&P obligations at each point in time.*

$$E_t i^*_{t,t+1} = E_t x_{t,t+1} \left(\frac{X_t}{X_t + L_t}\right) + i^l_{t,t+1} \left(\frac{L_t}{X_t + L_t}\right) \quad (18)$$

where, $E_t x_{t,t+1} = ln\ (E_t X_{t+1}/X_t)$ is the expected equity rate of return of the market portfolio in natural log terms. $i^l_{t,t+1}$ is the average rate of interest on S&P liabilities, calculated in section 3.2. $\frac{X_t}{X_t+L_t} nd \ \frac{L_t}{X_t+L_t}$ are the weights calculated in section 3.3 for the overall market's values of the equity X_t, and the liabilities $, L_t$.

3.6. Estimation of the default-free shadow real rate of return.

We wish to derive the expected shadow real rate of return, $E_t r^s_{t,t+1}$ utilizing a modified Fisherian paradigm, given that the default-free interest yields are represented by the U.S. Treasury yields:

$$E_t r^s_{t,t+1} = E_t i^*_{t,t+1} - (E_t \pi_{t,t+1} + IRP_{t+1}) - g_t\ , \quad (19)$$

$E_t \pi_{t,t+1}$ is the expected inflation at time *t*, and IRP_{t+1} is the *Inflation Risk Premium*, and $E_t i^*_{t,t+1}$ is the expected nominal rate of return at time *t*, derived in section 3.4, equation (19), implied from the market portfolio of the private sector. g_t is the business risk spread between the interest rate on default-free bonds and the value-weighted risk of bonds in the market portfolio, adjusted linearly by the relative weights in (18). There is no reason to assume that the weights are reflected differently in the spread of nominal bonds vs. the spread of inflation-protected bonds.

We assume that a positive correlation, if any, between business risk and inflation is already reflected in the IRP. Then, $E_t r^s_{t,t+1}$ is the expected default-free real rate of return at time *t*, labelled here as the *expected shadow default-free real rate of return.*

If we apply Mishkin's (1990) then the ex-post default-free real rate of return at time *t* is,

$$r^s_{t-1,t} = i_{t-1,t} - (\pi_{t-1,t} + IRP_t) - g_t \quad (20)$$

Decomposing the current yield into the expected rate and risk premia, or decomposing the BEI into expected inflation and IRP, as demonstrated in the affine term structure model,[18] or calibrating a

[18] *Finance theory introduced affine term structure models to derive the interest rate yield curve (Vasicek, 1977; Cox, Ingersoll, and Ross, 1985; Duffie & Kan, 1996).*

Stochastic Discount Factor (SDF), does not serve our purpose. We wish to derive the shadow default-free real rate of return, utilising relatively weak assumptions rather than calibrating a model to fit the data. Furthermore, for such decomposition, one must develop a model, somewhat unrelated to the focus of this paper, for the potential causes of inflation[19]. The literature still debates this issue. Some scholars attribute the post-pandemic rise in inflation to government expenditures following the pandemic and a spiral relation with wages. Stiglitz (2023), however, disagrees with the above interpretations, showing that the demand side is not the cause but rather supply-side technical factors. If we are not aiming for the decomposition of the yield into expected inflation and IRP we may utilise the following definition,

$$BEI_{t,t+1} \equiv (E_t\pi_{t,t+1} + IRP_{t+1}(E_t\pi_{t,t+1} + IRP_{t+1} \qquad \text{(D3)}$$

First, we generate daily expected inflation estimates through the changes in the annualised Break-Even Interest rate, $\Delta BEI_{t,t-12}$, i.e.,

$$\pi_{t,t-12} + \Delta BEI_{t,t-12}\ ^{20} \qquad (21)$$

where $\pi_{t,t-12}$ is the 12 months' actual inflation. Then, without loss of generality, we utilize the definition (D1) and replace $(E_t\pi_{t,t+1} + IRP_{t+1})$ in (18) with $BEI_{t,t+1}$,

$$E_t r^s_{t,t+1} = i^*_{t,t+1} - BEI_{t,t+1} - g_t\ . \qquad (22)$$

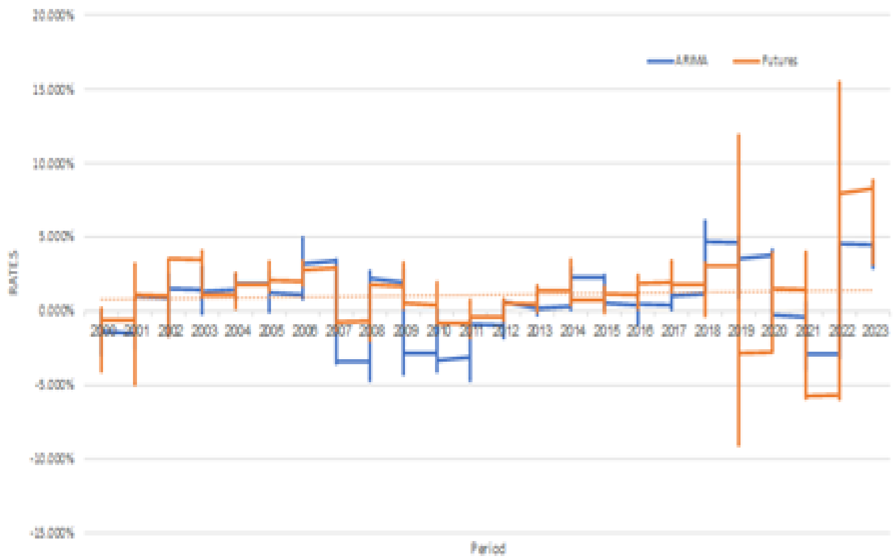
Seemingly, the shadow real rate of return is myopic, but given that the expected equity rate of return is based on the average maturity of S&P obligations. The end results based on the two methodologies are given in Figure 5.

Kim and Wright (2005), Cochrane and Piazzesi (2005), and Adrian, Crump, and Moench (2013) added an expectation component, thereby enhancing affine models' forecasting. Carr (2018), following Vasicek and using duration as a risk measure, calculates the expected interest rate along the risk dimension only.

[19] *Atkeson and Ohanian (2001), Stock and Watson (2009), and Kocherlakota (2016), argue that real variables like unemployment and output imbalances have little forecasting power for future inflation. See also Homer and Sylla (2005), Eichengreen (2015), and Hamilton et al. (2016).*

[20] *This alternative seems to be highly correlated with the inflation expectations survey conducted by the University of Michigan.*

Figure 5
The shadow default-free real rate of return (U.S. Treasuries)



Note: The shadow default-free real rate of return - measured in percentage term; orange line - based on the Futures data, and blue line - based on the ARIMA model.
Source: Raw data was extracted from FRED, St. Louis, and calculated as specified in the methodology

3.7. Stage B

The characteristics of the expected shadow default-free real rate of return will be verified by testing the following hypotheses.

- While the nominal and inflation-protected interest rates have exhibited a high and increasing correlation, the one between the *default-free real rate of return* and the nominal interest rate is low. If this hypothesis is confirmed, it lends itself to a different interpretation of the Fisherian paradigm.
- While the changes in the real GDP are not correlated with the inflation-protected real rate of interest, the changes in real GDP and the *expected shadow default-free real rate of return* are.
- While the variability pattern over the maturity of the inflation-protected real interest rate is not monotonic, the variability of the *expected shadow default-free real rate of return* over maturity is.

- While the *expected shadow default-free real rate of return* has exhibited no significant trend in the past twenty years, the inflation-protected interest rate, as demonstrated in the literature, has exhibited a declining trend.

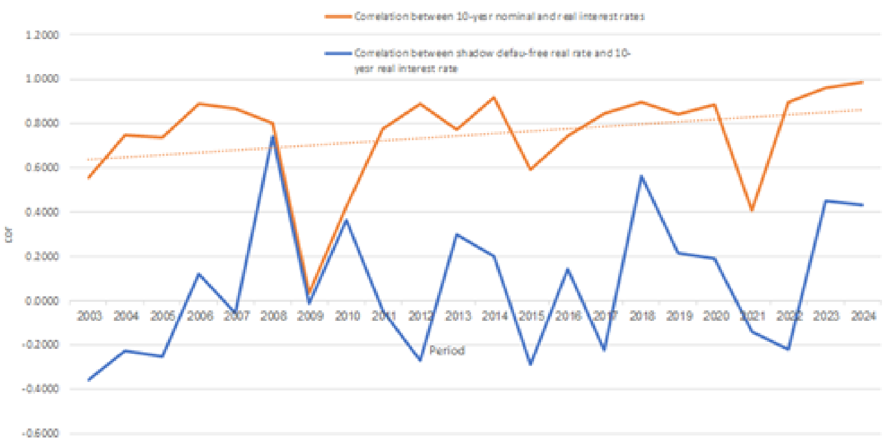
4. Results

4.1. The correlation between the default-free nominal rate of interest and the shadow default-free rate of return

Figure 6 presents the correlation between the default-free inflation-protected rates and the nominal interest rates. This correlation has increased significantly in the past few years. The graph also presents the correlation between the default-free nominal interest rate and the shadow default-free real rate of return. While the correlation between the 10-year maturity nominal and the real interest rates has risen significantly (from 0.73 to 0.88), the corresponding correlation between the nominal interest rate and the shadow real rate hovers around zero.

Figure 6

Correlation between real and nominal interest rates as opposed to the correlation between the shadow real and nominal rate (U.S. Treasuries 2003-2024)



Note: orange line - correlation between real and nominal interest rates; blue line - correlation between the shadow real and nominal rate.

Source: Data shown in Figure 5 and FRED, St. Louis

The Fisherian paradigm attributes the relative movements of the nominal rate, given the movements of the real rate, to the changes in expected inflation. Under this paradigm, changes in the nominal rates that stem from inflationary pressures but are followed by similar changes in the real rate seem counterintuitive. One may argue that real economic factors can affect both rates, but it is unlikely that during the entire period, the sole set of factors that affected both rates was the real ones, particularly in the past few years, when monetary factors clearly underlined the changes in the nominal rate. Additionally, one can argue that inflationary pressures have indirectly distorted real effects; however, it is unlikely that these effects would be of the same magnitude on both the nominal and real rates. Figure 6, besides conforming to the new version of the Fisherian paradigm, may support the view that the shadow real rate better represents the real rate rather than the inflation-protected rate.

4.2. Real GDP and the default-free shadow real rate of return

Following the results of the previous section, we suspect that the shadow default-free real rate of return will better reflect economic activity in real terms, represented by the behaviour of the real GDP. As widely stated in the literature, the inflation-protected interest rate does not reflect the behaviour of the real GDP. We confirm this result in Table 2a, which shows that the correlation between the real GDP and the inflation-protected interest rate, concurrently or with lags, is not significantly different from zero.

Table 2a

The correlation between real GDP and inflation-protected interest rates (2003-2024)

	5-year horizon	10-year horizon	20-year horizon
lag n=3	-0.035	-0.02895	-0.086784
lag n=2	-0.0506	-0.05985	-0.124975
lag n=1	-0.1099	-0.09358	-0.138323
Concurrent	-0.0929	-0.07938	-0.10972
lag n=-1	-0.0626	-0.05069	-0.086121
lag n=-2	-0.0475	-0.024	-0.053446
lag n=-3	-0.0097	-0.04649	-0.08135

Note: inflation-protected interest rates at time t, and real GDP with quarterly lags of 0 to 3 quarters.

Source of data: FRED, St. Louis

Alternatively, when we tested the correlation between the real GDP and the shadow default-free real rate, the correlation was significantly positive, particularly for the annualised, no-time-dimension shadow real rate. Table 2b presents the results.

Table 2b
The correlation between real GDP and the shadow rates of return (2003-2024)

	1	2	3	4	5	6
lag n=3	0.0218	-0.08978	-0.060434	0.120748	0.061766	0.134038
lag n=2	0.1113	0.012119	0.062615	0.119929	0.130862	0.141934
lag n=1	0.1895	0.133836	0.193069	0.222665	0.175331	0.134167
Concurrent	0.4403	0.336462	0.465163	0.309445	0.238143	0.123036
lag n=-1	0.3995	0.389534	0.483081	0.325674	0.233648	0.115161
lag n=-2	0.0838	0.363254	0.311460	0.275292	0.233820	0.111493
lag n=-3	-0.0838	0.295113	0.180900	0.247966	0.206063	0.047330

Notes: (1) - no time dimension, the shadow rate of return is based on Futures data; (2) - no time dimension, the shadow rate of return is based on the ARIMA model; (3) - no time dimension, the shadow rate of return is the average of (1) and (2); (4) - 5-year horizon; (5) - 10-year horizon; (6) - 20-year horizon. Quarterly lags of 0 to 3 quarters.

Source: Data presented in Figure 5 and FRED, St. Louis.

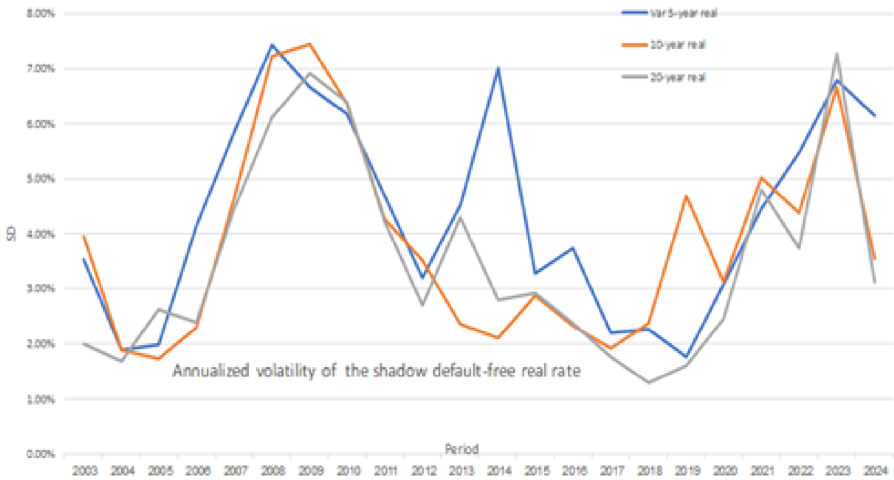
The results provide additional confirmation that the shadow default-free real rate of return better represents the real economy and conforms to a variant of the Fisherian paradigm.

4.3. The volatility pattern of the shadow default-free real rate of return

While Figure 3 presented a non-monotonic pattern of rate volatility over maturity for the inflation-protected interest rates, Figure 6 presented a monotonic pattern for the shadow default-free real rate of return over horizons of 5,10, and 20 years. However, as in the case of the inflation-protected rates in which the short-maturity rate has the highest volatility, we observe similar results for the shadow real rate; the shorter the horizon, the higher the volatility of the rate.

Figure 7

The pattern of volatility of the shadow default-free rate of return over various horizons (U.S. Treasuries, 2003-2024)



Note: The volatility is measured as the standard deviation of the default-free shadow rate of return. The standard deviation is calculated on the annualised daily shadow rates of return.

Source: rates presented in Figure 5.

A possible explanation for the non-monotonic behaviour of the volatilities of the inflation-protected interest rates may lie in the trading volume, as the market for the 20-year maturity bonds experiences lower volume. However, this may not be the proper explanation for the theoretical notion of the shadow real rates of return. Under the Keynesian paradigm, one would expect the volatility to be positively correlated with the maturities, but given that we observe an inverse yield curve (abnormal in Keynes' terms) during a significant portion of the sample period, the pattern shown in Figure 6 is consistent with the shape of an inverse yield curve.

4.4. Trend in the shadow expected default-free real rate of return

Recent views in the literature, as cited in the introduction, tend to conclude that the real interest rate exhibits a long-term declining trend. We argue that better enforcement and increased accessibility to information explain this trend over the past 100-300 years; however, we doubt that the decline in rates from the late 1980s until 2022 constitutes a long-term trend. Thus, we look at the behaviour of the shadow real rate of return.

Figure 5 clearly indicates that no consistent trend can be detected in the shadow default-free real rates of return for the past 20 years.

5. Conclusion

Two intriguing observations may be explained by substituting the default-free inflation-protected interest rates with the shadow default-free rate of return. The first intriguing observation is the high correlation between the inflation-protected and nominal interest rates. The second one is the observed low correlation between inflation-protected interest rates and real GDP growth rate. The shadow default-free rate of return is not directly observable. This paper suggested one estimation method of this rate, but we realise that the choice of estimation procedure is critical.

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